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FIXED-WEIGHT EIGENVALUE OPTIMIZATION OF TRUSS STRUCTURES BY SWARM INTELLIGENT ALGORITHMS

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ABSTRACT

Meta-heuristics have already received considerable attention in various engineering optimization fields. As one of the most rewarding tasks, eigenvalue optimization of truss structures is concerned in this study. In the proposed problem formulation the fundamental eigenvalue is to be maximized for a constant structural weight. The optimum is searched using Particle Swarm Optimization, PSO and its variant PSOPC with Passive Congregation as a recent meta-heuristic. In order to make further improvement an additional hybrid PSO with genetic algorithm is also proposed as PSOGA with the idea of taking benefit of various movement types in the search space. A number of benchmark examples are then treated by the algorithms. Consequently, PSOGA stood superior to the others in effectiveness giving the best results while PSOPC had more efficiency and the least fit ones belonged to the Standard PSO.

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KEY WORDS: structural sizing; eigenvalue optimization; genetic algorithm; particle swarm optimization; passive congregation

1. INTRODUCTION

Optimal structural design has been an active filed of research from 1904 up to now [1-4]. Nowadays application of optimal structural design is being extended to dynamic problems [5-16]. Structural designs and responses under dynamic loads highly depend on modal shapes and frequencies of the structure; that is solution of eigenvalue problem for optimal stiffness and/or mass matrices.

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Such an optimization problem is addressed by many investigators including pioneering work of Bellagamba et.al. [5-14]. Kaveh and Talatahari recently developed *Charged System Search* [15]. Kaveh and Zolghadr further compared results of its application with those reported by other existing methods for skeletal structures [16]. An interesting review of methods applied to truss weight minimization under eigenvalue constraints is presented by Grandhi at 1993 [17].

Two main classes of problem formulation can be distinguished in optimal design with eigenvalue variation. The most common case is the structural weight minimization under constraints on single or multiple eigenvalues; generally numbered starting from lower-energy modes of vibration. For this class of problems, some challenging points have already been reported [18]. The first one is switching between different modes when components' sizing or shape of the structure is being altered during the optimization. In the other hand, repeated eigenvalues exist in some types of structures like symmetric three-dimensional trusses. Tong et.al theoretically proved that a certain eigenvalue will not change if the structure undergoes uniform variation of mass or stiffness matrices [18]. They also studied solution existence of such eigenvalue problems stating that it is related to the fundamental natural frequency in truss structures.

The second formulation concerns the dual problem; that is searching for optimal (maximum) frequency of certain mode(s) for a fixed amount of structural material or weight.It is also addressed by some investigators [19].

The present work concerns the second class of formulation which bypasses many of the aforemaintioned challenges. Then Particle Swarm Optimization, PSO, as a vastly used meta-heuristic in both discrete and continuous global optimization is utilized for this problem [20-21]. PSO results on a number of benchmark problems are then evaluated and compared with one of its recent modified variants with Passive Congregation; i.e., PSOPC method [22]. A novel application of a hybrid genetic and swarm optimization called PSOGA [23] is also presented for this problem and compared with the other two methods. Theoretical discussion and numerical evaluation over the treated examples will declare the performance superiority among the three algorithms as follows.

2. PROBLEM FORMULATION

In many practical problems, the structural response to dynamic loading is mainly governed by the first frequency and vibration mode [17]. Hence, the first eigenvalue, λ_1, is considered here to be maximized via the following problem formulation:

$$Maximize \quad \lambda_1(\underline{X}) \tag{1}$$

$$Subject \quad to:$$

$$W(\underline{X}) - W^B = 0 \tag{2}$$

$$X_j^{LB} \le X_j \le X_j^{UB} \tag{3}$$

$$\det(\underline{K} - \lambda\ \underline{M}) = 0 \tag{4}$$

Whereas design variables include vector of X_j as the j^{th} member cross-section area limited to prescribed lower and upper bounds X_j^{LB}, X_j^{UB}, respectively. For every designed truss the corresponding fundamental eigenvalue λ_1 is the lowest root of Eq.(4) that addresses the lowest modal energy. $\underline{M}$ And $\underline{K}$ stand for the mass and stiffness matrices, respectively.

$W(\underline{X})$; I.e. the total structural weight is limited to a constant bound W^B and computed for every truss as:

$$W(\underline{X}) = \rho \sum X_j l_j \tag{5}$$

The constraints (3) and (4) are satisfied implicitly via programming the algorithm routine in this study. In order to handle the 1st equality constraint on structural weight a penalty approach is employed here which transforms the problem formulation into unconstrained form as:

$$Maximize \quad Fitness(\underline{X}) = \frac{\lambda_1}{1 + k_p C} \tag{6}$$

where C denotes the constraint violation and k_p stands for a penalty constant factor:

$$C = \left| \frac{W(\underline{X}) - W^B}{W^B} \right| \tag{7}$$

3. PARTICLE SWARM OPTIMIZATION

Swarm intelligence can be considered a base for many recent meta-heuristic and heuristic algorithms which deal with continuous or mixed discrete-continuous optimization problems [21]. As a primarily nature-inspired algorithm in this class, standard particle swarm optimization was introduced by Kennedy and Eberhart [20]. It mimics the principles used by birds' flock, synchronizing with each other during their move toward their goal. In PSO, a virtual bird is called a particle which makes every its movement as a vector-sum of the following three vectors called inertial, cognitive and social terms:

$$\underline{V}_i^{K+1} = C_w \underline{V}_i^K + r_c C_c (\underline{X}_i^{Pb} - \underline{X}_i^K) + r_s C_s (\underline{X}^{Gb} - \underline{X}_i^K) \tag{8}$$

And thus moves to its new position (new solution vector) $\underline{X}_i^{K+1}$ at the iteration $k+1$ as:

$$\underline{X}_i^{K+1} = \underline{V}_i^{K+1} + \underline{X}_i^K \tag{9}$$

The first velocity term in Eq.8 is oriented toward previous direction of movement; so called inertial term. The second direction is toward the best position of that i^{th} particle, $\underline{X}_i^{Pb}$; i.e. cognitive term while the term orientation is toward the global best position already

found by overall action of the entire swarm particles, $\underline{X}^{Gb}$; known as the social term. This formula indicates that each particle in PSO should keep in memory its two last movements and best of them. C_w is an inertial factor to control the influence of the previous velocity. In this study, C_w is linearly decreased from 0.9 to 0.5 during the search. C_c, C_s Stand for fixed magnifying coefficients scaled by random numbers r_c, r_s in range [0, 1] for inertial, cognitive and social terms, respectively.

4. ENHANCED SWARM ALGORITHMS

In 2004 He et.al introduced a new version of PSO [22]. They reviewed active and passive aggregation as well as passive and social congregation types of natural swarm behaviors and consequently introduced Particle Swarm Optimization with Passive Congregation, PSOPC with the following modified velocity relation:

$$\underline{V}_i^{K+1} = C_w \underline{V}_i^K + r_c C_c (\underline{X}_i^{Pb} - \underline{X}_i^K) + r_s C_s (\underline{X}^{Gb} - \underline{X}_i^K) + r_p C_p (\underline{R}_i - \underline{X}_i^K) \qquad (10)$$

In which a 4th term is added to Eq.8 including direction of the vector $\underline{R}_i$ and its corresponding bandwidth C_p and randomizer r_p. According to this presented passive congregation strategy each particle in the current swarm is affected by a random chosen design vector of the same swarm known as $\underline{R}_i$. He et.al stated that such a modification can be regarded a stochastic operator introducing perturbation into the search process [22]. However, since $\underline{R}_i$ does not contain any information out of the existing variables in the swarm. Hence, it may be considered a weak kind of exploitative operators.

In order to add more effectiveness to the standard PSO key features of *Genetic Algorithms* is concerned. GA has already been employed in several fields of engineering problem since it is first introduced by Holland [24]. Here, we introduce another modification to enhance standard PSO as follows:

$$\underline{V}_i^{K+1} = C_w \underline{V}_i^K + r_c C_c (\underline{X}_i^{Pb} - \underline{X}_i^K) + r_s C_s (\underline{X}^{Gb} - \underline{X}_i^K) + r_p C_p (\underline{X}_{GA}^k - \underline{X}_i^K) \qquad (11)$$

In this method known as PSOGA the 4th term indicates direction vector from the current position toward a new particle; $\underline{X}_{GA}^k$ which itself is obtained using the following subroutine:

- Copy all the current particles of the k^{th} swarm iteration into an auxiliary memory taken as the 1st genetic population
- Perform crossover with probability of *Pc* and then mutation with probability of *Pm* on the current population
- Perform tournament selection on fitness evaluated chromosomes of the population in the current generation and save the fittest one

- Repeat the last 2 steps for N_{GA} number of GA generations, and then announce the elitist chromosome among all of them as the particle $\underline{X}_{GA}^{k}$

Such a sub-algorithm employs *direct coding* so that every variable in a particle is analogous to a gene value in the corresponding chromosome [25]. In this regard, 1-point crossover and simple mutation by such an encoding scheme works quite well for the eigenvalue problem of the present study as will be shown in the next section.

The proposed PSOGA algorithm take benefit of both vector-sum jumps of swarm algorithms as in Eq.8 and powerful exploitation and exploration of genetic jumps over the search space. Therefore it is expected to enhance global search capability of the mentioned swarm algorithms; the matter is later confirmed by numerical tests, as well.

5. ALGORITHM PERFORMANCE MEASURES

Performance comparison between different optimization algorithms have been a challenging task in various engineering problems. Common terms to study in this issue are *efficiency* and *effectiveness* of the optimization method. Effectiveness means how close to the global optima the algorithms can get and escape from local optima. While the efficiency of an algorithm is related to how rapid it can converge. History curve of the best-so-far or elitist fitness found during iterations of the search is a common tool for this purpose.

When comparing two different algorithms on the same problem, it can be simply concluded that the one achieving the highest final fitness for certain number of iterations is more effective, provided that the history curves have the same initial fitness or population. In this paper a curve fitting strategy is employed to obtain an efficiency measure. Consider the elitist fitness history vs. search iteration number numerically derived during optimization and is to be fitted into an analytical function.

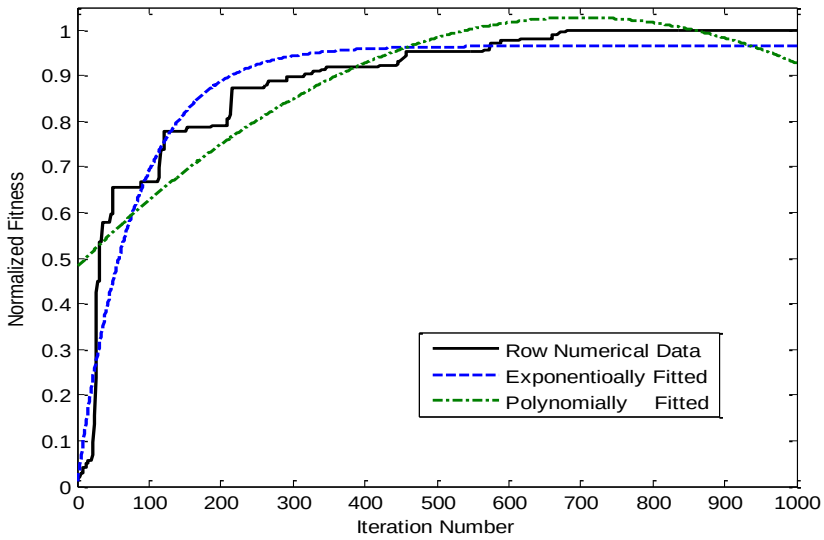


Figure 1.Comparison of polynomial and exponential curve-fitting for a sample convergence history

Figure 1 shows results of fitting such a curve once to a quadratic polynomial and then to an exponential function. As can be realized, the latter form has better agreement with fitness history end- and mid-points. Using a number of trial runs it was found that a sample PSO convergence for this problem can be fitted to a function of the form:

$$y = a(1 - e^{-bx}) \tag{12}$$

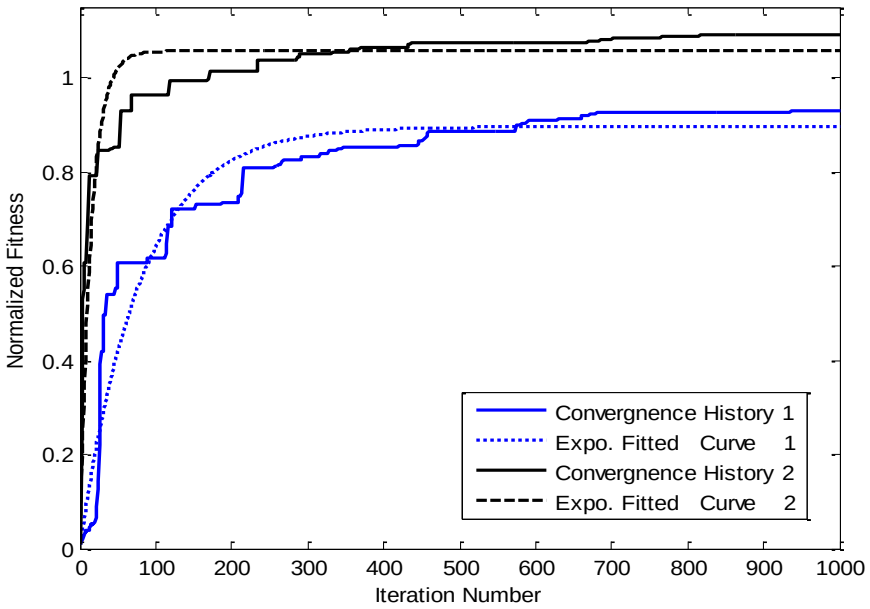


Figure 2.Variation of fitted-function curvature for two fitness histories showing different convergence rates

Figure 2 shows sample fitness histories generated by two different algorithms in bold lines and their corresponding fitted curves in dash-lines. The more the factor b is obtained, the quicker the convergence is. Such a factor is thus used here-in-after as a measure of convergence rate or algorithm efficiency. According to Equation 12 a scales the difference between elite fitness in the first and last iterations. Therefore it accounts for optimization effectiveness when the algorithms are started from the same population and its elitist fitness.

Another important issue is how to measure if the algorithm can provide proper diversity in its search agents up to the final convergence [25]. Lack of diversity in optimization may result in premature convergence to local optima or making difficulties in constraint handling which is a common challenge in structural problems. The present work employs some strategies to trace such a feature including mean value observation over swarm particles for parameters like fitness and objective function besides to definition of a variation measure as follows. Let σ be the difference between the lowest and the highest fundamental eigenvalue among the entire swarm in any iteration. Diversity of the swarm population will diminish as σ tends to zero and vice versa so it can be considered a measure of diversity in this study.

6. NUMERICAL INVESTIGATION

The aforementioned versions of swarm intelligent algorithms for the natural frequency maximization problem are tested here with some illustrative examples from literature. Every example has been solved for a number of trials to tune these parameters while only sample results are reported for the sake of conciseness. For the sake of true comparison, control parameters and the first randomly initiated population are kept the same between search algorithms in each example. In the present work, variation of the member cross-sections during the search is the source of stiffness matrix change that will consequently result in eigenvalue modification. The cross-section areas are varied by the algorithms continuously between X_j^{LB} and X_j^{UB} ; given for each case. A reference value, W_0, is also defined in each case as the total structural weight when every truss member is assigned its heaviest cross-section: X_j^{UB}. In every example the problem is solved with fixed-weight constraint of $W^B = \beta W_0$ for two distinct cases of $\beta_1 = 30\%$ and $\beta_2 = 75\%$.

Example 1: 10-bar truss

A 10-bar truss as depicted in Figure 3 is considered for this example as a sizing optimization benchmark [26]. Material density is taken 0.1 lb/in^2 and modulus of elasticity is $10000\,ksi$.

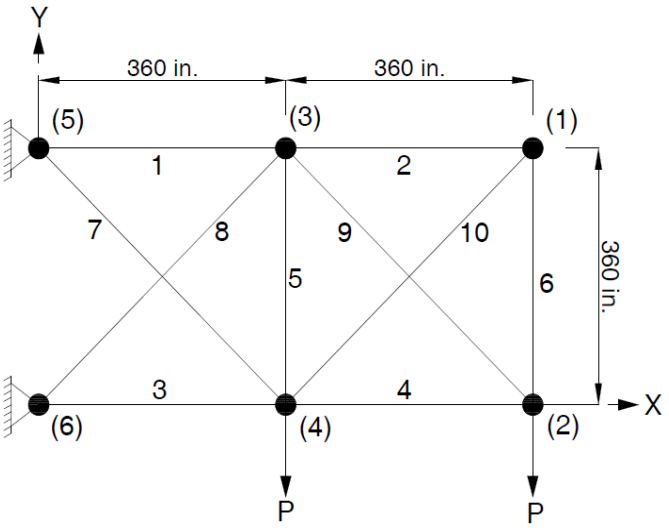


Figure 3.The 10-bar truss model and loading in example

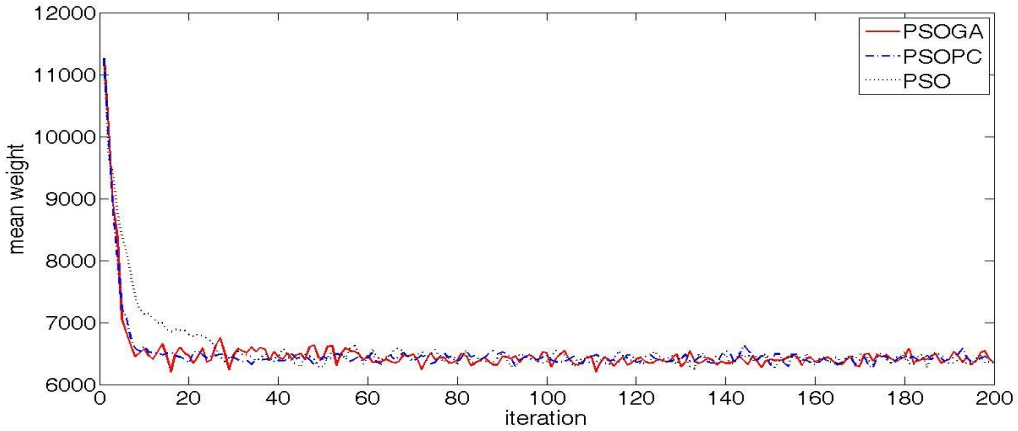
General and extra control parameters of PSOPC and PSOGA are given in Table1 and Table 2, respectively. Lower and upper bounds for member cross-sections are taken $X_j^{UB} = 0.1 in^2$ and $X_j^{UB} = 50.0 in^2$ resulting in the reference weight of $W_0 = 20982\,lb$. In this example, extra behavioral constraint is applied as member-stresses are limited to $\pm 25 ksi$ and the allowable nodal displacements are $\pm 2.0 in$ under the demonstrated loading condition with P of $100\,kips$.

Table 1: General optimization control parameters for 10-bar truss in example-1

Population Size	C_w	C_c	C_s	$Num.Iterations^{PSO}$
10	0.9 – 0.5	2	2	1000

Table 2: Extra parameters for PSOPC and PSOGA for 10-bar truss in example-1

C_p	Crossover Type	Crossover Probability	Mutation Probability	N_{GA}
1.5	1-point	0.85	0.15	2



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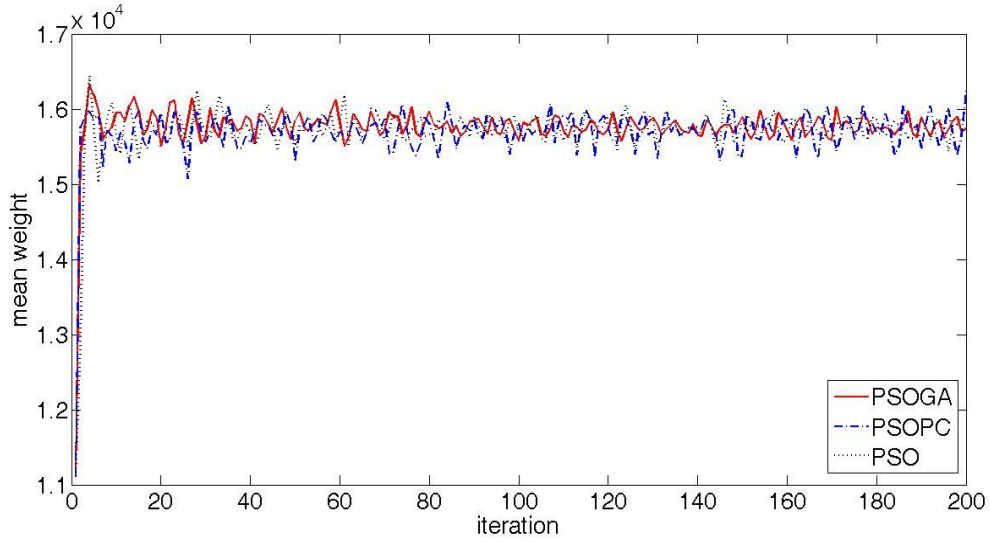


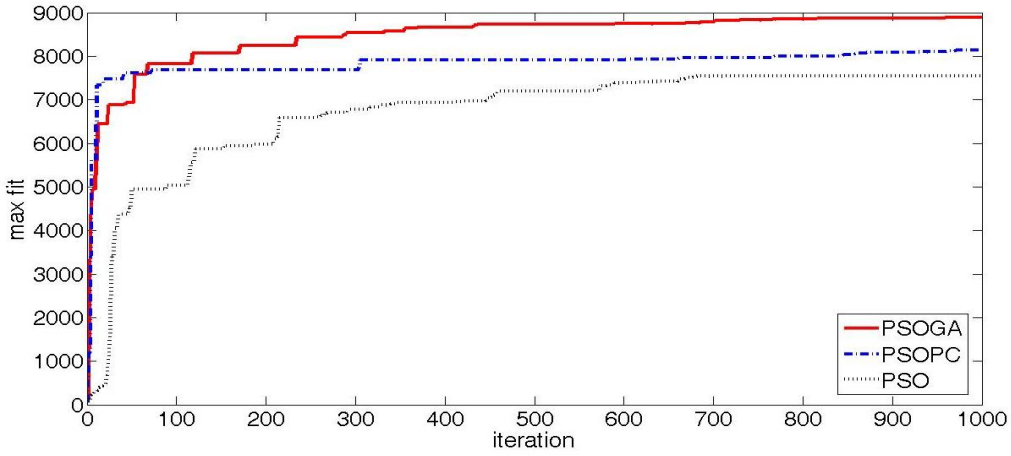
(b)

Figure 4. Mean structural weight of example-1 for (a) $\beta = 30\%$ and (b) $\beta = 75\%$

Figure 4 shows the average structural weight variation over population of the swarm

particles as the search progress. It can be noticed that for both β values, all the three algorithms gradually tend to satisfy the equality constraint; $W = \beta W_0$, however, PSOPC has more fluctuations around it.

History of the best achieved or elitist fitness and corresponding fundamental eigenvalue for $\beta = 30\%$ and $\beta = 75\%$ are depicted in Figures 5 and 6, respectively. It can be realized that for both cases PSOPC has achieved higher eigenvalues and is thus more effective than PSO. However, both PSO and PSOPC have been trapped in local optima in some β values resulting in lower fundamental eigenvalues than PSOGA. Note that regardless of the algorithm type, resulted history curves of the elitist fitness are smoother than those of the maximal fundamental eigenvalue in the presence of the constant weight constraint due to Equation 2.



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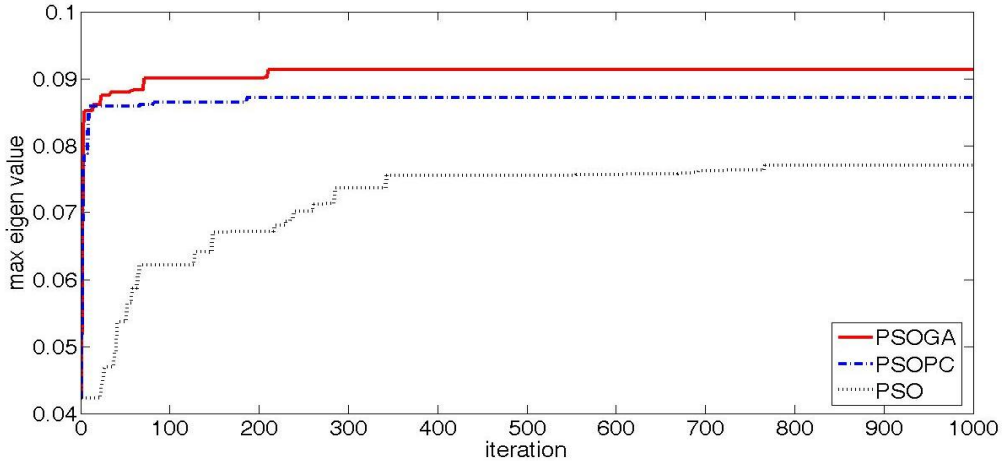
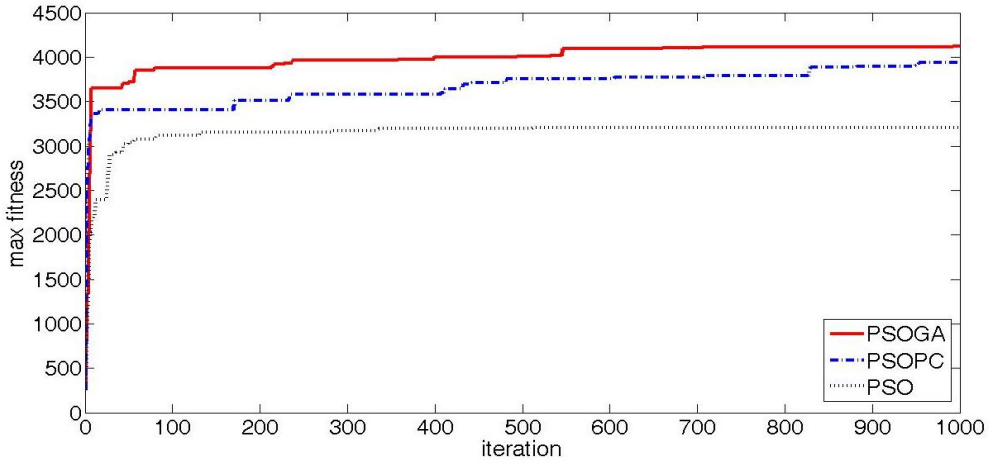


(b)

Figure 5. (a) The elitist fitness and (b) fundamental eigenvalue for $\beta = 30\%$ in example-1



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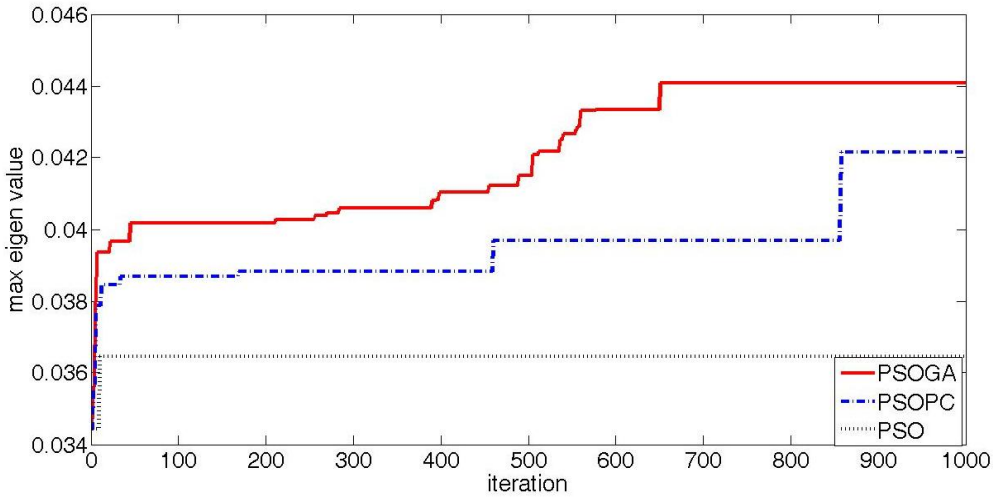
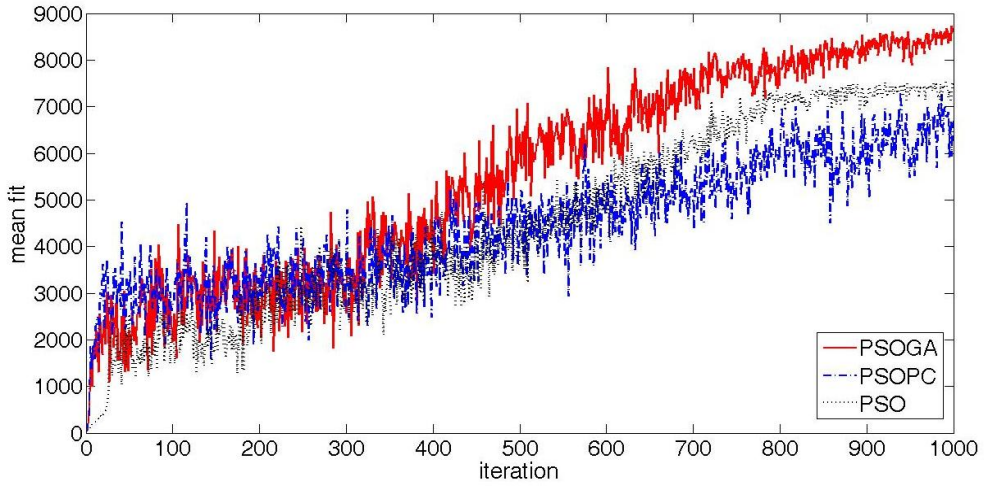


(b)

Figure 6. (a)The elitist fitness and (b) fundamental eigenvalue for $\beta = 75\%$ in example-1

In view of the mean fitness over the swarm particles in Figure 7, PSO over-rides PSOPC as the search progress but the most rewarding effectiveness still belongs to PSOGA.

It is notable in Figure 8 that PSOPC shows almost steady fluctuation of σ as a variation measure while in PSO it rapidly converges to zero. The matter confirms that the PSO rapidly tends to premature convergence while PSOPC suffers from relatively slow convergence rate. According to Figure 8, such a difference is magnified and more declared for $\beta_2 = 75\%$. In the other hand, for both β cases the σ of PSOGA exhibits a convergent trend but its fluctuation does not diminish as rapid as PSO. The matter confirms capability of PSOGA in taking benefit of suitable exploitative and explorative operators resulting in proper diversity of the population during the search.



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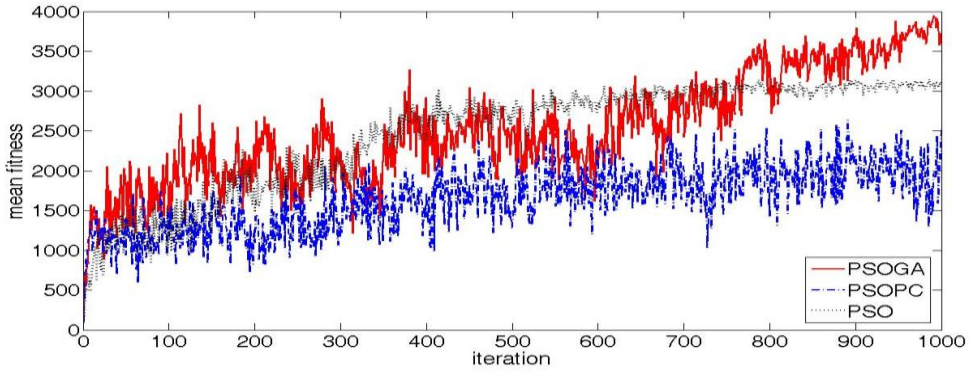


(b)

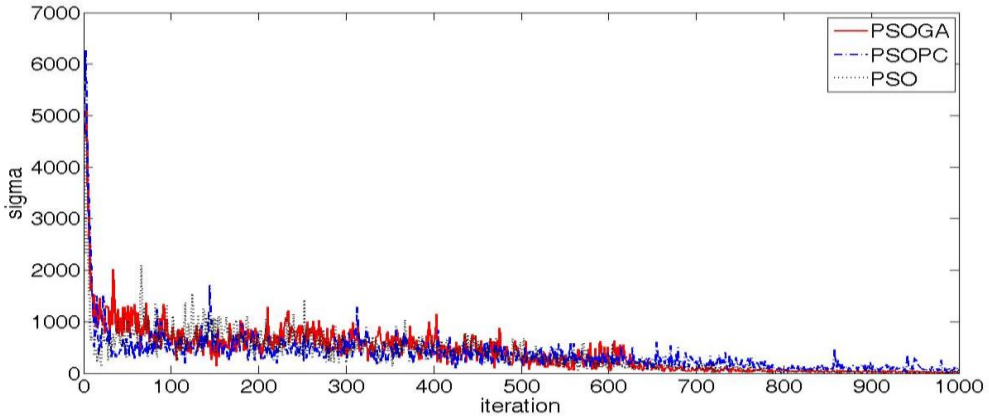
Figure 7. The mean fitness among swarm particles for (a) $\beta = 30\%$ and (b) $\beta = 75\%$ in example-1

Table 3: Performance index comparison of the treated algorithms for different β cases in example-1

Problem Case	Method	$10^{-3}a$	b
β_1	PSO	7.3	0.013
	PSOPC	7.9	**0.149**
	PSOGA	**8.6**	0.067
β_2	PSO	3.1	0.096
	PSOPC	3.6	**0.348**
	PSOGA	**4.0**	0.186

Considering Table 3, it is found for both β cases that PSOPC has the most b values and convergence speed while PSOGA leads to the most a/effectiveness measures. The matter

numerically confirms the above discussion.



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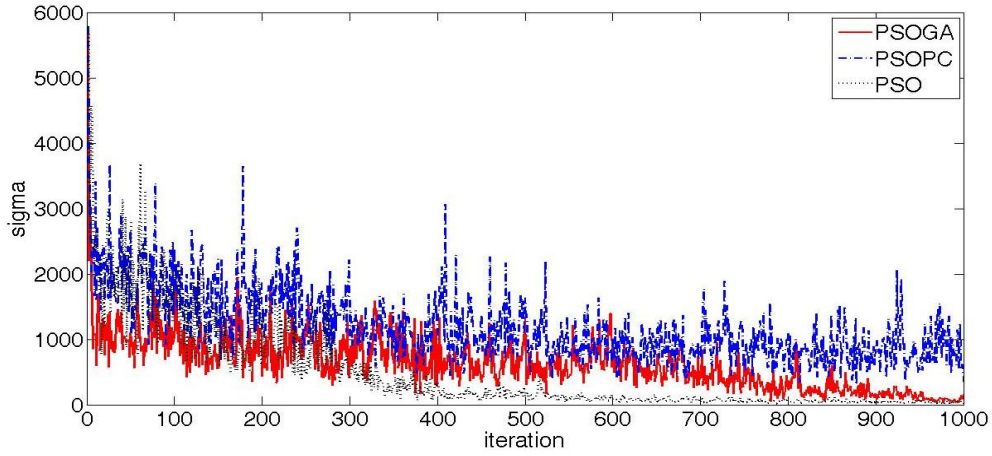


(b)

Figure 8.Trace of σ index for (a) β =30% and (b) β =75% in example-1

Example 2: 72-bar truss

In order to test the algorithms for three-dimensional case, a well-known 72-bar electrical transfer tower truss is considered as the 2nd example. The employed material for truss has a density of *2770kg/m*^{3} and elasticity modulus of *6.98×10*^{10}*Pa*. In this example the four upper nodes are assigned an additional non-structural mass of *2770kg* as depicted in Figure 9.

The cross-section area of any truss member can be assigned a floating-point value between $0.645cm^2$ and $30.000\ cm^2$, thus the reference weight is $W_0 = 18700.7kgf$ in this example. The problem formulation (6) is then applied to maximize the fundamental frequency of this truss under fixed weight constraint; $W = \beta W_0$ for two distinct cases: $\beta_1 = 30\%$ and $\beta_2 = 75\%$. General control parameters are taken the same as example-1 unless the number of total iterations which is reduced to 200. Extra parameters are given in Table 4.

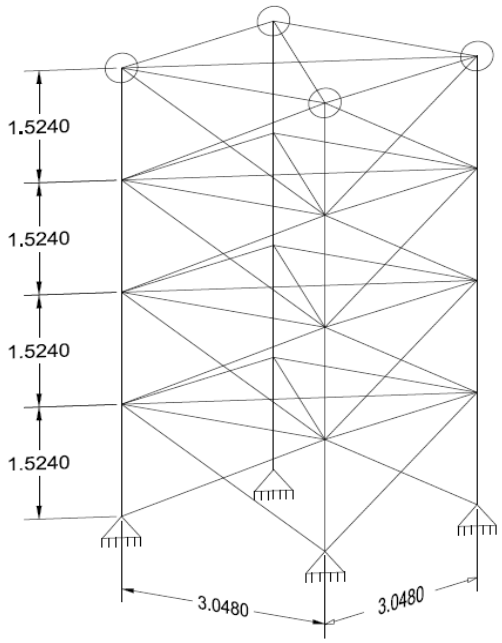
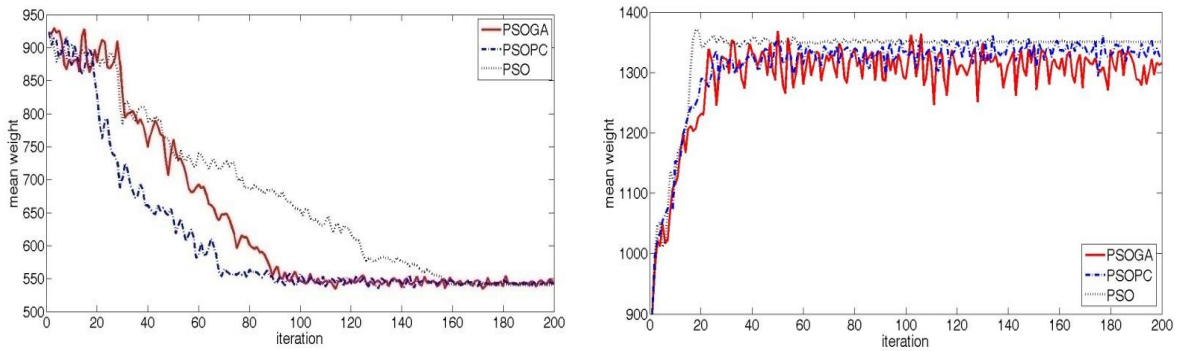


Figure 9.The 72-bar truss of example-2 (SI units)

Table 4: Extra control parameters for algorithms in 72-bar truss example

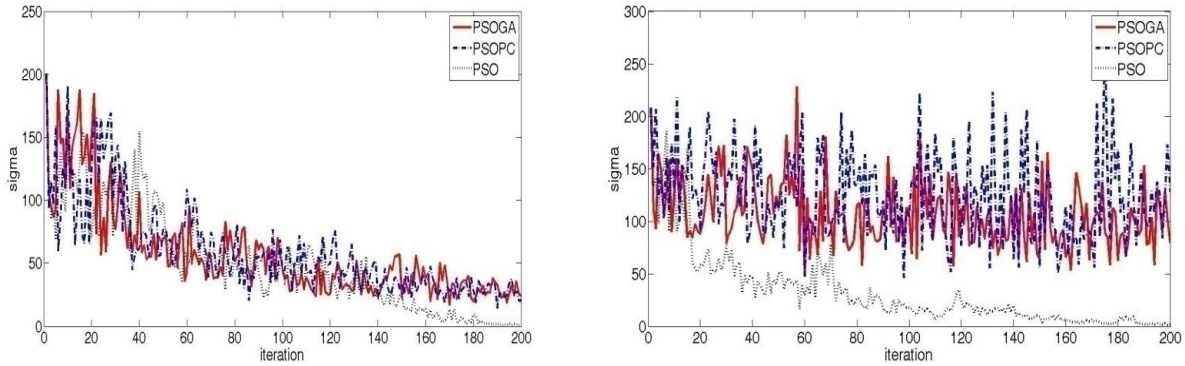
C_p	Crossover Type	Crossover Probability	Mutation Probability	N_{GA}
0.1	1-point	0.85	0.15	2

Figure 10 compares capability of the algorithms in handling the fixed-weight constraint. in this 3-dimensional example. The matter occurred more slowly for $\beta_1 = 30\%$ than the previous example; meanwhile PSOPC is better than PSOGA which itself is superior to PSO. For the case of $\beta_2 = 75\%$, however, all the algorithms have converged to the fixed weight in similar manner.



(a) (b)

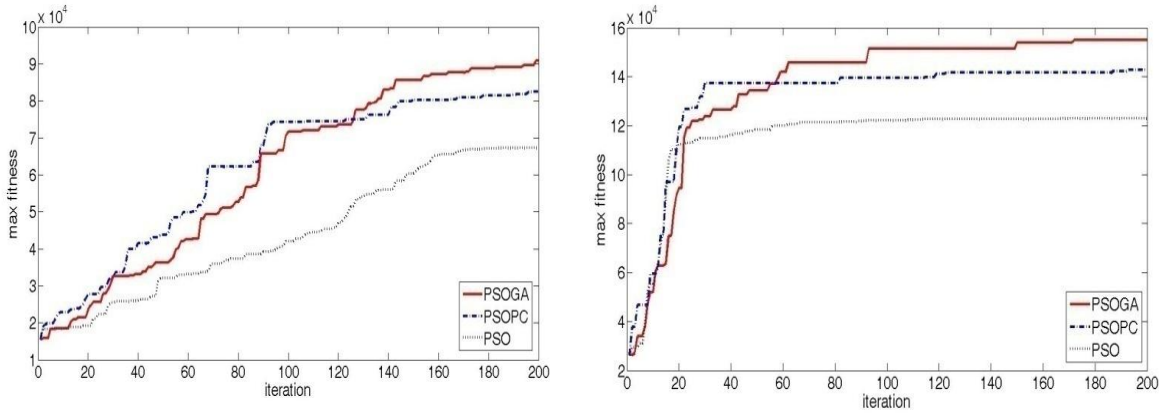
Figure 10. Mean structural weight of example-2 for (a) β =30% and (b) β =75%



(a) (b)

Figure 11. Trace of σ index for (a) β =30% and (b) β =75% in example-2

According to Figure 11, the diversity measure σ of PSOGA and PSOPC is higher than PSO for $\beta_1 = 30\%$ and the matter is more highlighted for the case of $\beta_2 = 75\%$. Such a case dependent and algorithm dependent difference in diversity can also be observed in view of the resulted fitness histories in Figure 12.



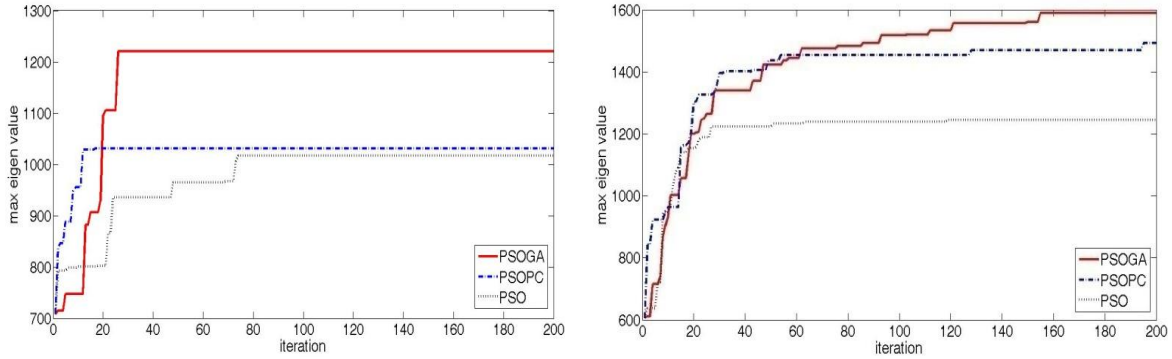
(a) (b)

Figure 12. History of elitist fitness in example-2 for (a) β =30% and (b) β =75%

Table 5 shows result of curve fitting to derive numerical performance indices. Like previous example, for both β cases again more convergence rates or b 's belong to PSOPC and then to PSOGA. However, such an arrange for achieving higher final fitness or effectiveness is changed to PSOGA as the best algorithm and then PSOPC and PSO, regarding a values. In overall view, values of a and b indices are also decreased with respect to the previous 2-dimensional example indicating more complexity of the current 3-dimentional example.

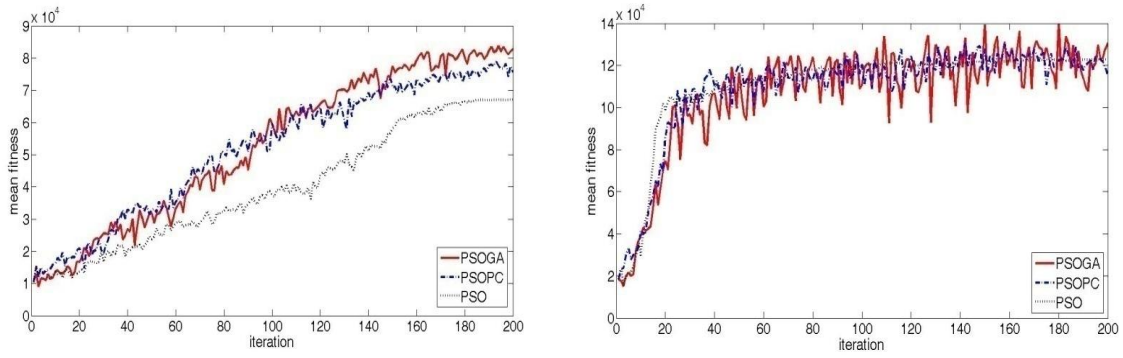
Table 5: Performance index comparison of treated algorithms for different β cases in example-2

Problem Case	Method	$10^{-3}a$	b
β_1	PSO	85	0.008
	PSOPC	86	**0.017**
	PSOGA	**110**	0.009
β_2	PSO	122	0.084
	PSOPC	141	**0.075**
	PSOGA	**152**	0.048



(a) (b)

Figure 13. Trace of maximum fundamental eigenvalue in example-2 for (a) β =30% and (b) β =75%



(a) (b)

Figure 14. Mean fitness of swarm population in example-2 for (a) β =30% and (b) β =75%

In order to concern effect of penalty function in constraint handling tracing diversity is extended here to the observation of maximum achieved fundamental eigenvalue in Figure 13. As can be seen, in case of $\beta_1 = 30\%$ the algorithms are relatively weaker to find newer maximal frequencies than for $\beta_2 = 75\%$ where PSOPC and specially PSOGA have shown

more desired and acceptable progress. According to Figure 14, this conclusion is further confirmed in tracing fluctuations in view of mean values for fitness over the swarm population during the search.

Example 3: 25-bar truss

A well-known 25-bar electrical transfer tower truss is considered for this example as depicted in Figure 15. Material density and modulus of elasticity are *0.1 lb/in*^{2} and $10000\,ksi$, respectively. Lower and upper bounds for member cross-sections are taken X_j^{lB}=*0.001in*^{2} and X_j^{UB}=*50.0in*^{2} resulting in the reference weight of W_0=*16536lb*. Control parameters are the same as previous example except $C_p$that is taken 0.5 and the algorithms are run for 2000 iterations.

Figure 16 shows capability of the employed method in constant-weight constraint handling. As can be realized PSOGA again has an acceptable capability while this time PSO has converged quicker than PSOPC. Such efficiency comparison is confirmed regarding b values in Table 6.

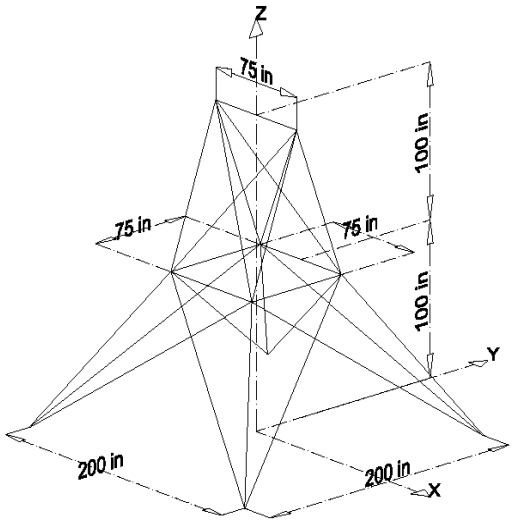
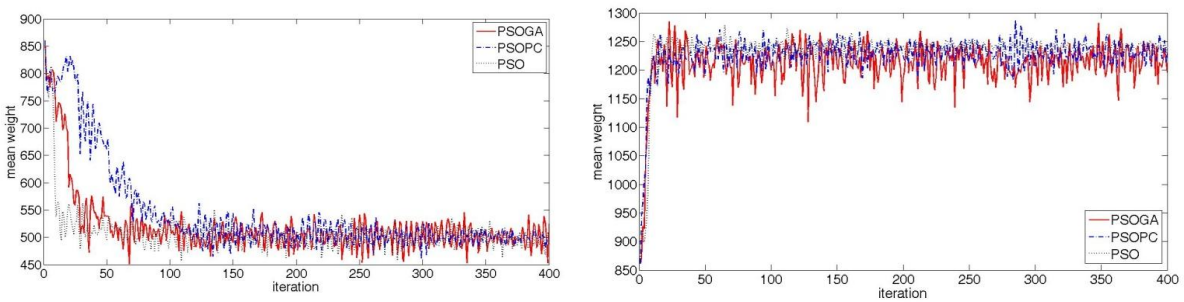


Figure 15. The 25-bar truss of example-3



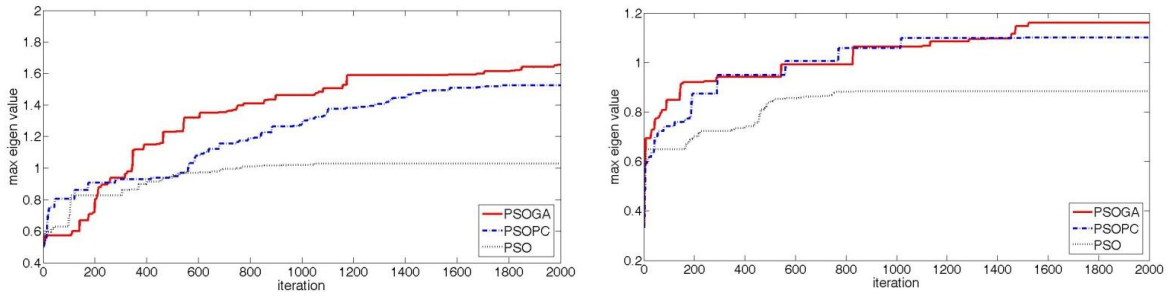
(a) (b)

Figure 16. Mean structural weight of example-3 for (a) $\beta = 30\%$ and (b) $\beta = 75\%$

Table 6: Performance index comparison of treated algorithms for different β cases in example-3

Problem Case	Method	a	b
β_1	PSO	102	**0.006**
	PSOPC	151	0.002
	PSOGA	**158**	0.003
β_2	PSO	85	0.014
	PSOPC	87	**0.032**
	PSOGA	**95**	0.029

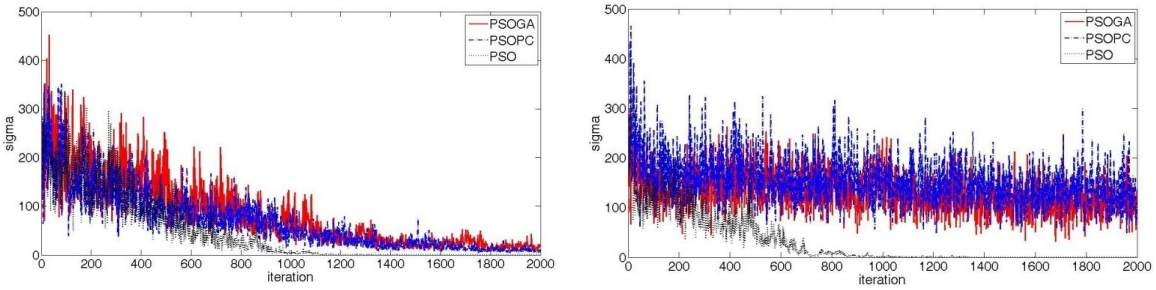
According to history of elitist fitness and fundamental eigenvalue in Figures 17, PSOGA again has the best rank in seeking global optimum while PSO has again led to premature convergence.



(a) (b)

Figure 15.Trace of maximum fundamental eigenvalue in example-3 for (a) β =30% and (b) β =75%

This matter is numerically confirmed by a values in Table 6 which is greatest for PSOGA and lowest for PSO. It can be also studied in view of population diversity index σ in Figure 18 that is rapidly tending zero for PSO while stays fluctuating for PSOPC and PSOGA. It is the reason to enable them escape from local optima toward global solution of the problem.



(a) (b)

Figure 16. Trace of σ index for (a) β =30% and (b) β =75% in example-2

7. CONCLUSION

In this paper fixed-weight structural eighenvalue optimization is studied utilizing several strategies to evaluate key issues in algorithms' performance analysis and comparison. Consequent implementation of a curve-fitting strategy with the proposed exponential function led to definition of a convergence rate b to evaluate efficiency and a scaling parameter a to determine effectiveness of the algorithms in addition to a diversity measure σ.

According to the achieved results, it is observed that PSO with the least σ and diversity have the most potential for being trapped in local optima. Applying passive congregation as in PSOPC has enhanced the algorithm efficiency resulted in more convergence rate than the other algorithms in the treated cases, however, it still showed potential of premature convergence. That is due to the fact that no new information is entered to the swarm by random re-use of current swarm positions during passive congregation. In the other hand, PSOGA takes merit of genotypic exploitation and mutation to provide the best diversity among the studied algorithms. Consequently, it has given optima with the highest quality resulting in greater values for a index. In view of the average values for parameters like fitness or modal frequency, PSOGA has shown superior performance over PSO and PSOPC because it is neither too fast in convergence to lose required diversity nor too fluctuating about the mean to loose the recently found optimum. Therefore, PSOGA can be recommended for the proposed eigenvalue optimization problem due to its better effectiveness and higher quality of results.

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REFERENCES

1. Michell AGM. The limits of economy in frame structures. *Philosoph Mag Sect 6*, 1904; **8**(47): 589-97.
2. Rozvany GIN. *Optimal Design of Flexural Systems: Beams, Grillages, Slabs, Plates, and Shells*, Pergamon Press, Oxford, 1976.
3. Ringertz UT. A branch and bound algorithm for topology optimization of truss structures, *Eng Opt*, 1986; **10**:111-24.
4. Yang RJ, Chuang CH. Optimal topology design using linear programming, *Comput Struct*, 1994; **52**(2): 265–75.
5. Bellagamba L, Yang TY. Minimum mass truss structures with constraints on fundamental natural frequency, *AIAA J*, 1981; **19** (11): 1452-58.
6. Arora JS, Dutta A. Explicit and implicit methods for design sensitivity analysis of nonlinear structures under dynamic loads, *Appl Mech Rev*, 1997, **50** (11): 11-19.
7. Grandhi RV, Venkayya VB. Structural optimization with frequency constraints, *AIAA J*, 1988; **26**(7): 858–66.
8. Sadek E. An optimality criterion method for dynamic optimization of structures, *Int J Numer Meth Engrng*, 1989; **28**: 579-92.

9. Masad JA. Optimization methods with structural dynamics applications, *Comput & Struct*, 1997; **62**(3): 521-525.
10. Yang XY, Xie YM, Steven GP, Querin OM. Topology optimization for frequencies using an evolutionary method, *J Struct Eng*, 1999; **125**(12): 1432-38.
11. Sedaghati R, Suleman A, Tabarrok B. Structural optimization with frequency constraints usingfinite element force method. *AIAA J*, 2002; **40**(2): 382-88.
12. Lingyun W, Mei Z, Guangming W, Guang M. Truss optimization on shape and sizing with frequency constraints based on genetic algorithm, *J Comput Mech*, 2005; **25**: 361-68.
13. Gholizadeh S, Salajegheh E, Torkazadeh P. Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network, *J Sound Vib*, 2008; **312**: 316-31.
14. Gomes MH. Truss optimization with dynamic constraints using a particle swarm algorithm, *Expert Sys Appl*, 2011; **38**: 957–68.
15. Kaveh A, Talatahari S. Novel heuristic optimization method: charged system search, *Acta Mech*, 2010; **213** (3-4): 267-89.
16. Kaveh A, Zolghadr A. Shape and size optimization of truss structures with frequency constraints using enhanced charged system search algorithm, *Asian J Civil Eng*, 2011; **12**:487-509.
17. Grandhi RV, Structural optimization with frequency constraints – a review, *AIAA J*, 1993; **31**(12): 2296-303.
18. Tong WH, Jiang JS, Liu GR. Solution existence of the optimization problem of truss structures with frequency constraints, *Int J Solids and Struct*, 2000; **37**:4043-60.
19. Mijar AR, Swan CC, Arora JS, Kosaka I. Continuum topology optimization for concept design of frame bracing systems , *J Struct Eng*, 1998: **124**, 541-50.
20. Kennedy J, Eberhart R. Particle Swarm Optimization, *Proceedings of the International Conference on Neural Networks*, 1995; Perth, Australia, 1942-48.
21. Kennedy J, Eberhart, R, *Swarm Intelligence*, Academic Press, London, 2001.
22. He S, Wu KH, Yen JY, Sanders JR, Paton RC. A particle swarm optimizer with passive congregation, *Bio Systems*, 2004;**78**:135-47.
23. Shahrouzi M. A new hybrid genetic and swarm optimization for earthquake accelerogram scaling, *Int J Optim Civil Eng*, 2011; **1**(1): 127-40.
24. Holland J, *Adaptation in natural and artificial systems, an introductory analysis with application to biology, control and artificial intelligence*. Ann Arbor, The University of Michigan Press; 1975.
25. Kaveh A, Shahrouzi M. Dynamic selective pressure using hybrid evolutionary and ant system strategies for structural optimization, *Int J Numer Methods Eng*, 2008; **73** (4), 544-63.
26. Kaveh A, Zolghadr A. Truss optimization with natural frequency constraints using a hybridized CSS-BBBC algorithm with trap recognition capability, *Comput Struct*, 2012; **102-103**, 14-27.