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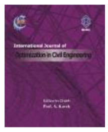

COST OPTIMIZATION OF END-FILLED CASTELLATED BEAMS USING META-HEURISTIC ALGORITHMS

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ABSTRACT

The main object of this research is to optimize an end-filled castellated beam. In order to support high shear forces close to the connections, sometimes it becomes necessary to fill certain holes in web opening beam. This is done by inserting steel plates and welding from both sides. Optimization of these beams is carried out using three meta-heuristic methods involves CSS, CBO, and CBO-PSO algorithms. To compare the performance of these algorithms, the minimum cost of the beam is taken as the design objective function. Also, in this study, two common types of laterally supported castellated beams are considered as design problems: beams with hexagonal openings and beams with circular openings. A number of design examples are considered to solve in this case. Comparison of the optimal solution of these methods demonstrates that the hexagonal beams have less cost than cellular beams. It is observed that optimization results obtained by the CBO-PSO for more design examples have less cost in comparison to the results of the other methods.

Keywords: meta-heuristic methods, optimum design, end-filled castellated beams, hexagonal opening beam, cellular opening beam, cost.

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1. INTRODUCTION

Since the 1940's, the production of structural beams with higher strength and lower cost has been an asset to engineers in their efforts to design more efficient steel structures. Due to the limitations on maximum allowable deflections, using of section with heavy weight and high

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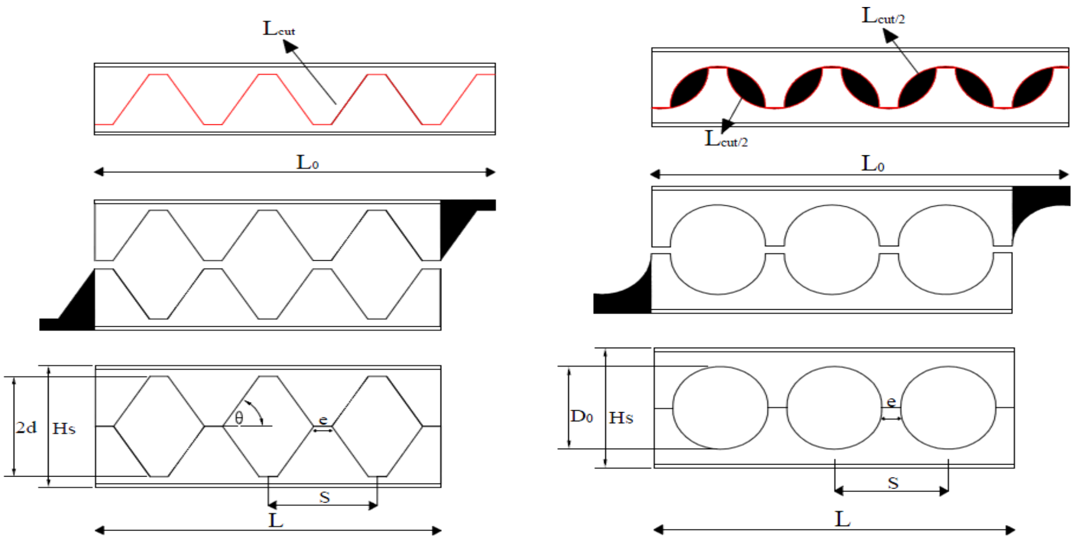
capacity in the design problem cannot always be utilized to the best advantage. As a result, several new methods have been created for increasing the stiffness of steel beams without increase in the weight of steel required. Castellated beam is one of them that become basic structural elements within the design of building, like a wide-flange beam [1].

Castellated beams are varieties of girders that are manufactured by cutting a hot rolled beam lengthwise using computer control plasma arc torches, often in half-circle or half-hexagon patterns Fig. 1. The split halves are then offset and welded back together to form a deeper beam with full circular or hexagonal shaped web openings. Cellular beams, which contain circular openings, are currently the most widely used perforated beams due to their beneficial weight-to-stiffness ratio, and the ability to pass services. The resulting holes in the webs permit mechanical ducts, plumbing, and electrical lines to pass through the beam rather than beneath the beam. Web-openings have been used for many years in structural steel beams in a great variety of applications because of the necessity and economic advantage. The principle advantage of the steel beam castellation process is that designer can increase the depth of a beam to raise its strength without adding steel. The resulting castellated beam is approximately 50% deeper and much stronger than the original unaltered beam [2-6].



(a) (b)

Figure 1. (a) A castellated beam with hexagonal opening; (b) A castellated beam with circular opening

In practice, in order to support high shear forces close to the connection or for reasons of fire safety, sometimes it becomes necessary to fill certain openings. In cellular beams, this is done by inserting discs made of steel plates and welding from both sides Fig. 2.

There are different meta-heuristic optimization methods; Genetic Algorithms (GA) [7], Ant Colony Optimization (ACO) [8], Harmony Search algorithm (HS) [9], Particle Swarm Optimizer (PSO) [10], Charged System Search method (CSS) [11], Bat algorithm [12], Ray optimization algorithm (RO) [13], Krill-herd algorithm [14], Dolphin Echolocation algorithm (DE) [15], Colliding Bodies Optimization (CBO) [16] are some of such meta-heuristic algorithms, see also Kaveh [17]. The CSS algorithm is developed by Kaveh and Talatahari [11] and has been successfully utilized in many optimization problems. Charged System Search is a population-based search method, where each agent (CP) is considered as a charged sphere with radius a, having a uniform volume charge density which can produce an electric force on the other CPs.

The colliding bodies optimization method (CBO) is one of the recently developed meta-heuristic algorithms that utilizes simple formulation and it requires no parameter tuning. This algorithm is based on one-dimensional collisions between two bodies, where each agent solution is modeled as a body [16, 17].

One of the recently hybrid meta-heuristics is the CBO-PSO algorithm, presented by the authors [18]. This algorithm is made by combining the PSO and the CBO methods, where the positive features of PSO are added to CBO algorithms to enhance the efficiency of the approach.

The main objective of the present study is to optimize the cost of castellated beams with end-filled openings. Thus, three above mentioned approaches are used to design of beams with circular and hexagonal holes.

The present paper is organized as follows: In the next section, the design of castellated beam is introduced. In section 3, statement of the optimization design problem is formulated, based on the Steel Construction Institute Publication Number 100 and Euro code3. In Section 4, the algorithms are briefly introduced. In Section 5, the cost of end-filled castellated beam as the design objective function is minimized, and finally Section 6 concludes the paper.

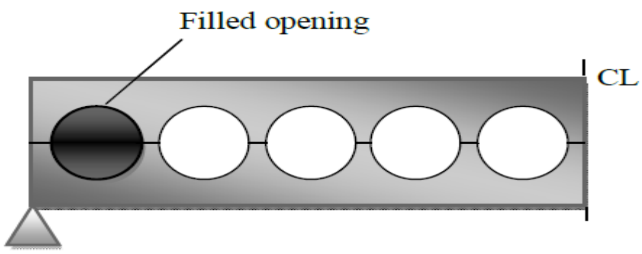


Figure 2. Example of beam with filled opening

2. DESIGN OF CASTELLATED BEAMS

The theory behind the castellated beam is to reduce the weight of the beam and improved stiffness by increased moment of inertia resulting from increased depth without adding additional material. Due to the presence of holes in the web, the structural behavior of castellated steel beam is different from that of the standard beams. At present, there is not a prescribed design method due to the complexity of the behavior of castellated beams and their associated modes of failure [2]. The strength of a beam with various web opening is determined by considering the interaction of the flexure and shear at the openings. There are many failure modes to be considered in the design of a beam with web opening consisting of lateral- torsional buckling, Vierendeel mechanism, flexural mechanism, rupture of welded joints and web post buckling. Lateral- torsional buckling may occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. In this paper it is assumed the compression flange of the castellated beam is restrained by the floor system. Therefore, the overall buckling strength of the castellated beam is omitted from the design consideration. These modes are closely associated with beam geometry, shape parameters, type of loading, and provision of lateral supports. In the design of castellated beams, these criteria should be considered [19-25]:

2.1. Overall beam flexural capacity

This mode of failure can occur when a section is subjected to pure bending. In the span subjected to pure bending moment, the tee-sections above and below the openings yields in a manner similar to that of a standard webbed beam. Therefore, the maximum moment under applied external loading, should not exceed the plastic moment capacity of the castellated beam [2, 21].

$$M_U \leq M_P = A_{LT} P_Y H_U \tag{1}$$

where A_{LT} is the area of lower tee, P_Y is the design strength of steel, and H_U is distance between center of gravities of upper and lower tees.

2.2. Beam shear capacity

In the design of castellated beams, it is necessary to control two modes of shear failures. The first one is the vertical shear capacity and the upper and lower tees should undergo that. Sum of the shear capacity of the upper and lower tees are checked using the following equations:

$$\begin{aligned} P_{VY} &= 0.6 P_Y \left(0.9 A_{WUL}\right) && \text{circular opening} \\ P_{VY} &= \frac{\sqrt{3}}{3} P_Y \left(A_{WUL}\right) && \text{hexagonal opening} \end{aligned} \tag{2}$$

The second one is the horizontal shear capacity. It is developed in the web post due to the change in axial forces in the tee-section as shown in Fig. 3. Web post with too short mid-depth welded joints may fail prematurely when horizontal shear exceed the yield strength. The horizontal shear capacity is checked using the following equations [2, 21]:

$$\begin{aligned} P_{VH} &= 0.6 P_Y \left(0.9 A_{WP}\right) && \text{circular opening} \\ P_{VH} &= \frac{\sqrt{3}}{3} P_Y \left(A_{WP}\right) && \text{hexagonal opening} \end{aligned} \tag{3}$$

where A_{WUL} is the total area of the webs of the tees and A_{WP} is the minimum area of web post.

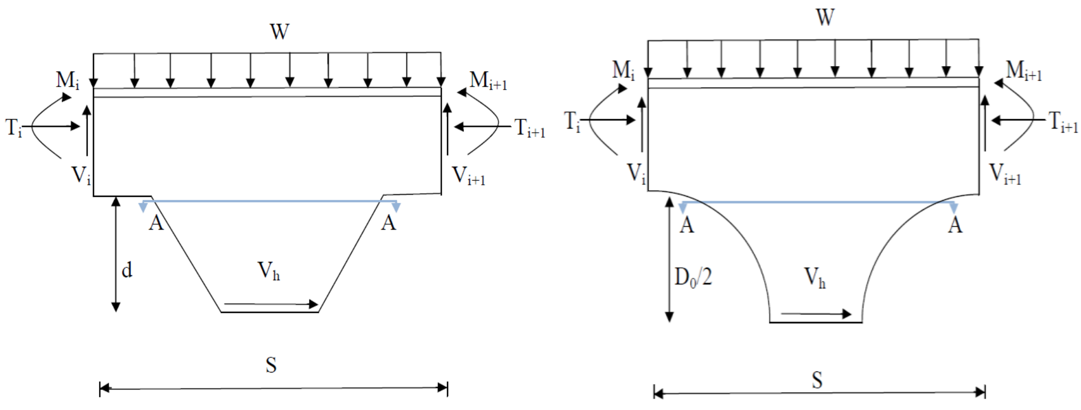


Figure 3. Horizontal shear in the web post of castellated beams, (a) hexagonal opening, (b) circular opening

2.3. Flexural and buckling strength of web post

As above mentioned, it is assumed that the compression flange of the castellated beam restrained by the floor system. Thus the overall buckling of the castellated beam is omitted from the design consideration. The web post flexural and buckling of capacity in castellated beam is given by [2, 21]:

$$\frac{M_{MAX}}{M_E} = [C_1.\alpha - C_2.\alpha^2 - C_3] \tag{4}$$

where M_{MAX} is the maximum allowable web post moment and M_E is the web post capacity at critical section A-A shown in Fig. 3. C_1, C_2 and C_3 are constants obtained by following expressions:

$$C_1 = 5.097 + 0.1464(\beta) - 0.00174(\beta)^2 \tag{5}$$

$$C_2 = 1.441 + 0.0625(\beta) - 0.000683(\beta)^2 \tag{6}$$

$$C_3 = 3.645 + 0.0853(\beta) - 0.00108(\beta)^2 \tag{7}$$

where $\alpha = \frac{S}{2d}$ for hexagonal openings, and $\alpha = \frac{S}{D_0}$ for circular openings, also $\beta = \frac{2d}{t_w}$ for hexagonal openings, and $\beta = \frac{D_0}{t_w}$ for circular openings, S is the spacing between the centers of holes, d is the cutting depth of hexagonal opening, D_0 is the holes diameter and t_w is the web thickness.

2.4. Vierendeel bending of upper and lower tees

Vierendeel mechanism is always critical in steel beams with web openings, where global shear force is transferred across the opening length, and the Vierendeel moment is resisted by the local moment resistances of the tee-sections above and below the web openings.

Vierendeel bending results in the formation of four plastic hinges above and below the web opening. The overall Vierendeel bending resistance depends on the local bending resistance of the web-flange sections. This mode of failure is associated with high shear forces acting on the beam. The Vierendeel bending stresses in the circular opening obtained by using the Olander’s approach Fig. 4. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as follows [21]:

$$\frac{P_0}{P_U} + \frac{M}{M_P} \le 1.0 \tag{8}$$

where P_0 and M are the force and the bending moment on the section, respectively. P_U is equal to area of critical section $\times P_Y$, M_P is calculated as the plastic modulus of critical section $\times P_Y$ in plastic section or elastic section modulus of critical section $\times P_Y$ for other sections.

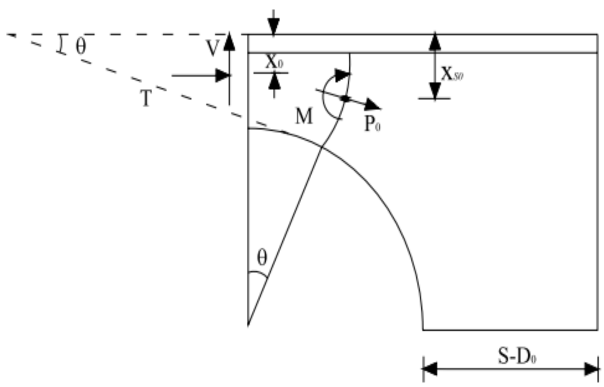


Figure 4. Olander's curved beam approach

The plastic moment capacity of the tee-sections in castellated beams with hexagonal opening are calculated independently. The total of the plastic moment is equal to the sum of the Vierendeel resistances of the above and below tee-sections [2]. The interaction between Vierendeel moment and shear forces should be checked by the following expression:

$$V_{OMAX}.e - 4M_{TP} \le 0 \tag{9}$$

where V_{OMAX} and M_{TP} are the maximum shear force and the moment capacity of tee-section, respectively.

2.5. Deflection of castellated beam

Serviceability checks are high importance in the design, especially in beams with web opening where the deflection due to shear forces is significant. The deflection of a castellated beam under applied load combinations should not exceed span/360. In castellated beams with circular opening, the deflection at each point is calculated by following expression:

$$Y_{TOT} = Y_{MT} + Y_{WP} + Y_{AT} + Y_{ST} + Y_{SWP} \tag{10}$$

where Y_{MT} ,Y_{WP} ,Y_{AT} ,Y_{ST} and Y_{SWP} are deflection due to bending moment in tee , deflection due to bending moment in web post of beam, deflection due to axial force in tee, deflection due to shear in tee and deflection due to shear in web post, respectively. These equations are provided in Ref. [21].

For a castellated beam with hexagonal opening and length L subjected to transverse loading, the total deflection is composed by two terms: the first term corresponds to pure moment action f_b, and the second one corresponds to shear action f_s. Thus, the total deflection can be calculated by the following expression:

$$f = f_b + f_s = c_1 L^3 + c_2 L \tag{11}$$

c_1 and c_2 are determined by means of a curve fitting technique [23].

3. OPTIMIZATION OF END-FILLED CASTELLATED BEAM

The main initiative for producing and using castellated beam is to suppress the cost of material by applying more efficient cross sectional shapes made from standard profiles in combination with aesthetic and architectural design considerations. In a castellated beam, there are many factors that require special considerations when estimating the cost of beam, such as man-hours of fabrication, weight, price of web cutting and welding process. At this study, it is assumed that the costs associated with man-hours of fabrication for hexagonal and circular opening are identical. Thus, the objective function includes three parts: The beam weight, price of the cutting and price of the welding. In the end-filled case, the price of plates is added to the total cost. Therefore, the objective function can be expressed as:

$$\mathrm{F}_{\mathrm{cost}} = \rho(A_{\mathrm{initail}}(L_0) + 2A_{hole}.t_w).p_1 + L_{cut}.p_2 + (L_{weld}).p_3 \tag{12}$$

where p_1, p_2 and p_3 are the price of the weight of the beam per unit weight, length of cutting and welding for per unit length, L_0 is the initial length of the beam before castellation process, ρ is the density of steel, A_{initail} is the area of the selected universal beam section, A_{hole} is the area of a hole, P_{hole} is the perimeter of a hole, L_{cut} and L_{weld} are the cutting length and welding length, respectively. The length of cutting is different for hexagonal and circular web-openings. The dimension of the cutting length is described by following equations:

For circular opening,

$$L_{\text{cut}} = \pi D_0.NH + 2e(NH + 1) + \frac{\pi D_0}{2} + e + 2.P_{hole} \tag{13}$$

For hexagonal opening,

$$L_{cut} = 2NH\,(e + \frac{d}{\sin(\theta)}) + 2e + \frac{d}{\sin(\theta)} + 2.P_{hole} \tag{14}$$

where NH is the total number of holes, e is the length of horizontal cutting of web, D_0 is the diameter of holes, d is the cutting depth, θ is the cutting angle, and P_{hole} is the perimeter of hole related to filled opening.

Also, the welding length for both of circular and hexagonal openings is determined by Eq. (15).

$$L_{weld} = e(NH + 1) + 4.P_{hole} \tag{15}$$

L_{cut} is shown for both of circular and hexagonal openings in Fig. 1.

3.1. Design of castellated beam with circular opening

Design process of a cellular beam consists of three phases: The selection of a rolled beam, the selection of a diameter, and the spacing between the center of holes or total number of holes in the beam as shown in Fig. 1 [21, 22]. Hence, the sequence number of the rolled beam section in the standard steel sections tables, the circular holes diameter and the total number of holes are taken as design variables in the optimum design problem. The optimum design problem formulated by considering the constraints explained in the previous sections can be expressed as the following:

Find an integer design vector $\{X\} = \{x_1, x_2, x_3\}^T$ where x_1 is the sequence number of the rolled steel profile in the standard steel section list, x_2 is the sequence number for the hole diameter which contains various diameter values, and x_3 is the total number of holes for the cellular beam [21]. Hence the design problem can be expressed as:

Minimize Eq. (12)

Subjected to

$$g_1 = 1.08 \times D_0 - S \le 0 \tag{16}$$

$$g_2 = S - 1.60 \times D_0 \le 0 \tag{17}$$

$$g_3 = 1.25 \times D_0 - H_S \le 0 \tag{18}$$

$$g_4 = H_S - 1.75 \times D_0 \le 0 \tag{19}$$

$$g_5 = M_U - M_P \le 0 \tag{20}$$

$$g_6 = V_{MAXSUP} - P_V \le 0 \tag{21}$$

$$g_7 = V_{OMAX} - P_{VY} \le 0 \tag{22}$$

$$g_8 = V_{HMAX} - P_{VH} \le 0 \tag{23}$$

$$g_9 = M_{A-AMAX} - M_{WMAX} \le 0 \tag{24}$$

$$g_{10} = V_{TEE} - 0.50 \times P_{VY} \le 0 \tag{25}$$

$$g_{11} = \frac{P_0}{P_U} + \frac{M}{M_P} - 1.0 \le 0 \tag{26}$$

$$g_{12} = Y_{MAX} - L/360 \le 0 \tag{27}$$

where t_W is the web thickness, H_S and L are the overall depth and the span of the cellular beam, and S is the distance between centers of holes. M_U is the maximum moment under the applied loading, M_P is the plastic moment capacity of the cellular beam, V_{MAXSAP} is the maximum shear at support, V_{OMAX} is the maximum shear at the opening, V_{HMAX} is the maximum horizontal shear, M_{A-AMAX} is the maximum moment at A-A section shown in Fig. 3. M_{WMAX} is the maximum allowable web post moment, V_{TEE} represent the vertical shear on the tee at $\theta = 0$ of web opening, P_0 and M are the internal forces on the web section as shown in Fig. 4, and Y_{MAX} denotes the maximum deflection of the cellular beam [21,25].

3.2. Design of castellated beam with hexagonal opening

In design of castellated beams with hexagonal openings, the design vector includes four design variables: The selection of a rolled beam, the selection of a cutting depth, the spacing between the center of holes or total number of holes in the beam and the cutting angle as shown in Fig. 3. Hence the optimum design problem formulated by the following expression:

Find an integer design vector $\{X\} = \{x_1, x_2, x_3, x_4\}^T$ where x_1 is the sequence number of the rolled steel profile in the standard steel section list, x_2 is the sequence number for the cutting depth which contains various values, x_3 is the total number of holes for the castellated beam and x_4 is the cutting angle. So, the design problem turns out to be as follows:

Minimize Eq. (12)
Subjected to

$$g_1 = d - \frac{3}{8}.(H_S - 2t_f) \le 0 \tag{28}$$

$$g_2 = (H_S - 2t_f) - 10 \times (d_T - t_f) \le 0 \tag{29}$$

$$g_3 = \frac{2}{3}.d.\cot\phi - e \le 0 \tag{30}$$

$$g_4 = e - 2d.\cot\phi \le 0 \tag{31}$$

$$g_5 = 2d.\cot\phi + e - 2d \le 0 \tag{32}$$

$$g_6 = 45^\circ - \phi \le 0 \tag{33}$$

$$g_7 = \phi - 64^\circ \le 0 \tag{34}$$

$$g_8 = M_U - M_P \le 0 \tag{35}$$

$$g_9 = V_{MAXSUP} - P_V \le 0 \tag{36}$$

$$g_{10} = V_{OMAX} - P_{VY} \le 0 \tag{37}$$

$$g_{11} = V_{HMAX} - P_{VH} \le 0 \tag{38}$$

$$g_{12} = M_{A-AMAX} - M_{WMAX} \le 0 \tag{39}$$

$$g_{13} = V_{TEE} - 0.50 \times P_{VY} \le 0 \tag{40}$$

$$g_{14} = V_{OMAX}.e - 4M_{TP} \le 0 \tag{41}$$

$$g_{15} = Y_{MAX} - L/360 \le 0 \tag{42}$$

where t_f is the flange thickness, d_T is the depth of the tee-section, M_P is the plastic moment capacity of the castellated beam, M_{A-AMAX} is the maximum moment at A-A section shown in Fig. 3, M_{WMAX} is the maximum allowable web post moment, V_{TEE} represent the vertical shear on the tee, M_{TP} is the moment capacity of tee-section and Y_{MAX} denotes the maximum deflection of the castellated beam with hexagonal opening [2].

4. OPTIMIZATION ALGORITHMS

In this paper, three algorithms are considered that all of them are meta-heuristic methods. These algorithms start with a set of randomly selected candidate solutions of the optimization problem and attempt to improve the quality of the set based on a cost function. A summary of these methods are described in the following subsections.

4.1. Charged system search

Charged System Search (CSS), proposed by Kaveh and Talatahari [11], has its governing rules inspired from electrostatics and Newtonian mechanics. It is a population-based search approach, where each agent (CP) is considered as a charged sphere with radius *a*, having a uniform volume charge density which can produce an electric force on the other CPs. The force magnitude for a CP located in the inside of the sphere is proportional to the separation distance between the CPs, while for a CP located outside the sphere it is inversely proportional to the square of the separation distance between the particles. The resultant forces or acceleration and the motion laws determine the new location of the CPs. The pseudo-code for the CSS algorithm can be summarized as follows:

Step 1: *Initialization*. The initial positions of CPs are determined randomly in the search space and the initial velocities of charged particles are assumed to be zero. The values of the fitness function for the CPs are determined and the CPs are sorted in an increasing order. A number of the first CPs and their related values of the fitness function are saved in a memory, so called charged memory (**CM**).

Step 2: *Determination of forces on CPs.* The force vector is calculated for each CP as

$$F_J = \sum_{i,i\neq j}\left(\frac{q_i}{a^3}.r_{i,j}.i_1 + \frac{q_i}{r_{i,j}^{\;2}}.i_2\right) ar_{i,j} P_{i,j}\left(X_i - X_j\right) \quad \left\langle \begin{array}{l} i_1 = 1, i_2 = 0 \Leftrightarrow r_{i,j} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{i,j} \geq a \\ j = 1,2,\ldots,N \end{array} \right. \tag{43}$$

where F_J is the resultant force acting on the j th CP; N is the number of CPs. The magnitude of charge for each CP (q_i) is defined considering the quality of its solution as

$$q_i = \frac{fit(i) - fitworst}{fitbest - fitworst}, i = 1,2,\ldots,N \tag{44}$$

where $fitbest$ and $fitworst$ are the best and the worst fitness of all particles, respectively; $fit(i)$ represents the fitness of the agent i; and N is the total number of CPs. The separation distance $r_{i,j}$ between two charged particles is defined as follows:

$$r_{i,j} = \frac{\left\| X_i - X_j \right\|}{\left\| \left(X_i - X_j\right)/2 - X_{best} \right\| + \varepsilon} \tag{45}$$

where X_i and X_j are respectively the positions of the i th and j th CPs, X_{best} is the position of the best current CP, and ε is a small positive number. Here, $P_{i,j}$ is the probability of moving each CP towards the others and is obtained using the following function:

$$P_{i,j} = \begin{cases} 1 & \dfrac{fit(i) - fitbest}{fit(j) - fit(i)} > rand \wedge fit(j) < fit(i) \\ 0 & else \end{cases} \tag{46}$$

In Eq. (43), $ar_{i,j}$ indicates the kind of force and is defined as

$$ar_{i,j} = \begin{cases} 1 & rand > 0.80 \\ 0 & else \end{cases} \tag{47}$$

where $rand$ represents a random number.

Step 3: *Solution construction*. Each CP moves to the new position and the new velocity is calculated by:

$$X_{j,new} = rand_{j,1}.K_a.F_j + rand_{j,2}.K_V.V_{j,old} + X_{j,old} \tag{48}$$

$$V_{j,new} = X_{j,new} - X_{j,old} \tag{49}$$

where K_a is the acceleration coefficient; K_v is the velocity coefficient to control the influence of the previous velocity; and $rand_{j,1}$ and $rand_{j,2}$ are two random numbers uniformly distributed in the rang (0,1); and K_a and K_v are taken as

$$K_a = 0.5 \times \left(1 + \frac{iter}{iter_{\max}}\right), K_v = 0.5 \times \left(1 - \frac{iter}{iter_{\max}}\right) \tag{50}$$

where $iter$ is the iteration number and $iter_{\max}$ is the maximum number of iterations.

Step 4: *Updating process.* If a new CP exits from the allowable search space, a harmony search-based handling approach is used to correct its position. In addition, if some new CP vectors are better than the worst ones in the **CM**; these are replaced by the worst ones in the **CM**.

Step 5: *Termination criterion control.* Steps 2-4 are repeated until a termination criterion is satisfied.

4.2 Colliding bodies optimization algorithm

The Colliding bodies optimization algorithm is one of the meta-heuristic search methods that recently developed. It is a population-based search approach, where each agent is considered as a colliding body (CB) with mass m. The idea of the CBO algorithm is based on observation of a collision between two objects in one dimension; in which one object collides with another object and they move toward minimum energy level [16, 17].

In the CBO algorithm, each solution candidate X_i is considered as a colliding body (CB). The massed objects are composed of two main equal groups; i.e. stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects; (ii) to push stationary objects towards better positions. After the collision, the new positions of the colliding bodies are updated based on their new velocities.

The pseudo-code for the CBO algorithm can be summarized as follows:

Step 1: *Initialization*. The initial positions of CBs are determined randomly in the search space:

$$x_i^0 = x_{\min} + rand.(x_{\max} - x_{\min}) \quad i = 1, 2, ..., n \tag{51}$$

where x_i^0 determines the initial value vector of the ith CB. $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable values vectors of variables, respectively; rand is a random number in the interval [0,1]; and n is the number of CBs.

Step 2: *Determination of the body mass for each CB.* The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \quad k = 1,2,...,n \tag{52}$$

where $fit(i)$ represents the objective function value of the agent i ; n is the population size. Obviously a CB with good values exerts a larger mass than the bad ones. Also, for maximizing the objective function the term $\frac{1}{fit(i)}$ is replaced by $fit(i)$.

Step 3: *Arrangement of the CBs.* The arrangement of the CBs objective function values is performed in ascending order (Fig. 5a). The sorted CBs are equally divided into two groups:

The lower half of CBs (stationary CBs); These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0 \quad i = 1,2,...,\frac{n}{2} \tag{53}$$

The upper half of CBs (moving CBs): These CBs move toward the lower half. Then, according to Fig. 5b, the better and worse CBs, i.e. agents with upper fitness value of each group will collide together. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_i - x_{i-\frac{n}{2}} \quad i = \frac{n}{2}+1,...,n \tag{54}$$

where v_i and x_i are the velocity and position vector of the ith CB in this group, respectively; $x_{i-\frac{n}{2}}$ is the ith CB pair position of x_i in the previous group.

Step 4: *Calculation of the new position of the CBs.* After the collision, the velocity of bodies in each group is evaluated using collision laws and the velocities before collision. The velocity of each moving CB after the collision is:

$$v'_i = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}} \quad i = \frac{n}{2}+1,...,n \tag{55}$$

where v_i and v'_i are the velocity of the ith moving CB before and after the collision, respectively; m_i is the mass of the ith CB; $m_{i-\frac{n}{2}}$ is mass of the ith CB pair. Also, the velocity of each stationary CB after the collision is:

$$v'_i = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}} \quad i = 1,\ldots,\frac{n}{2} \tag{56}$$

where $v_{i+\frac{n}{2}}$ and v_i are the velocity of the ith moving CB pair before and the ith stationary CB after the collision, respectively; m_i is mass of the ith CB; $m_{i+\frac{n}{2}}$ is mass of the ith moving CB pair. As mentioned previously, ε is the coefficient of restitution (COR) and for most of the real objects, its value is between 0 and 1. It defined as the ratio of the separation velocity of two agents after collision to the approach velocity of two agents before collision. In the CBO algorithm, this index is used to control of the exploration and exploitation rate. For this goal, the COR is decreases linearly from unit to zero. Thus, ε is defined as:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{57}$$

where $iter$ is the actual iteration number and $iter_{\max}$ is the maximum number of iterations, with COR being equal to unit and zero representing the global and local search, respectively.

New positions of CBs are obtained using the generated velocities after the collision in position of stationary CBs.

The new positions of each moving CB is:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v'_i \quad i = \frac{n}{2}+1,\ldots,n \tag{58}$$

where x_i^{new} and v'_i are the new position and the velocity after the collision of the ith moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of the ith stationary CB pair. Also, the new positions of stationary CBs are obtained by:

$$x_i^{new} = x_i + rand \circ v'_i \quad i = 1,\ldots,\frac{n}{2} \tag{59}$$

where x_i^{new}, x_i and v'_i are the new position, old position and the velocity after the collision of the ith stationary CB, respectively. $rand$ is a random vector uniformly distributed in the range (-1,1) and the sign ‘‘∘’’ denotes an element-by-element multiplication.

Step 5: *Termination criterion control.* Steps 2-4 are repeated until a termination criterion is satisfied.

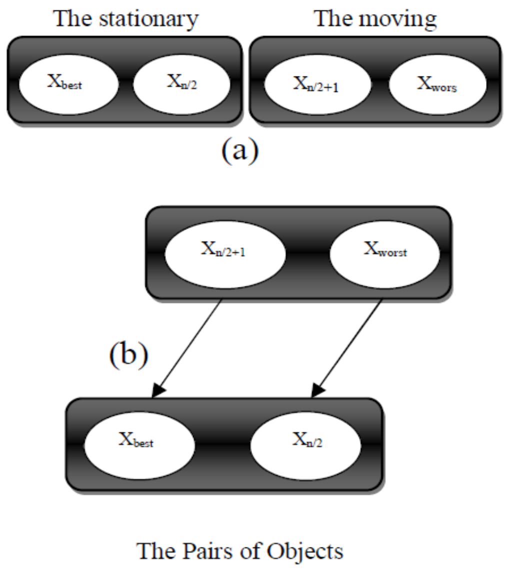


Figure 5. (a) The sorted CBs in an increasing order. (b) The pairs of objects for the collision

4.3 The CBO-PSO method

This hybrid CBO and PSO algorithm, so called CBO-PSO has been proposed by Kaveh and Shokohi [18] to improve the performance of the standard CBO. Both of these methods, (CBO and PSO) are population-based algorithms and find optimum solutions by changing the position of the agents. However, the movement strategies are different for the CBO and PSO. The PSO algorithm utilizes the local best and the global best to determine the direction of the movement, while the CBO approach uses the collision laws and the velocities before collision to determine the new positions. Using of the local best and the global best are the main reasons for the success of PSO. However, in spite of having the above-mentioned advantages, the standard PSO is infamous of premature convergence. This algorithm has some problems in controlling the balance between the exploration and exploitation due to ignoring the effect of other agents.

Similar to the PSO method, the CBO algorithm uses the previous velocities, when the upper half of CBs (moving CBS) move toward the stationary CBs. As it is mentioned in the previous section, after the collision, the velocity of all CBs is evaluated using the velocity of moving CBs before the collision and the mass of the paired CBs. This will lead to the loss of the best position of particles which is found in previous iteration. Therefore, in the present hybrid algorithm, the advantages of the PSO consist of the local best, and the global best are added to the CBO algorithm. For this purpose, the best position of the stationary particles is saved in a memory called stationary bodies memory (SBM). Also, another memory is considered to save the better position of each particle that up to this point has been found so far. This memory, so- called particles memory (PM), is treated as the local best in the PSO, and it is updated by following expression:

$$PM_{k+1}^{i} = \begin{cases} PM_{k}^{i} \rightarrow F(X_{k+1}^{i}) \geq F(X_{k}^{i}) \\ X_{k+1}^{i} \rightarrow F(X_{k+1}^{i}) \leq F(X_{k}^{i}) \end{cases} \tag{60}$$

in which the first term identifies that when the new position is not better that the previous one, the updating will not be performed, while when the new position is better than the so far stored good position, the new solution vector is replaced. With the above definitions, and considering the above- mentioned new memories, the velocity of CBs after collision are modified by following equations:

For moving particles,

$$v'_i = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}} + r_1 c_1.(PM_i - x_i) + r_2 c_2.(SPM - x_i) \quad i = \frac{n}{2}+1,...,n \tag{61}$$

For stationary particles,

$$v'_i = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}} + r_1 c_1.(PM_i - x_i) + r_2 c_2.(SPM - x_i) \quad i = 1,...,\frac{n}{2} \tag{62}$$

5. DESIGN EXAMPLES

In this section, in order to compare fabrication cost of the end-filled castellated beams with circular and hexagonal holes, three beams are selected. Among the steel section list of British Standards 64 Universal Beam (UB) sections starting from $254\times102\times28$ UB to $914\times419\times388$ UB are chosen to constitute the discrete set for steel sections from which the design algorithm selects the sectional designations for the castellated beams. In the design pool of holes diameters 421 values are arranged which varies between 180 and 600 mm with increment of 1 mm. Also, for cutting depth of hexagonal opening, 351 values are considered which varies between 50 and 400 mm with increment of 1 mm and cutting angle changes from 45 to 64. Another discrete set is arranged for the number of holes. Likewise, in all the design problems, the modulus of elasticity is equal to205 kN/mm^2 and Grade 50 is selected for the steel of the beam which has the design strength 355 MPa. The coefficients P_1, P_2 and P_3 in the objective function are considered 0.85, 0.30 and 1.00, respectively [26, 27].

5.1 Castellated beam with 4-m span

A simply supported beam with 4m span shown in Fig. 6 is selected as the first design example. The beam is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 kN also acts at mid-span of the beam and the allowable displacement of the beam is limited to 12 mm. The number of CBs is taken as 50 and maximum number of iterations is considered 200.

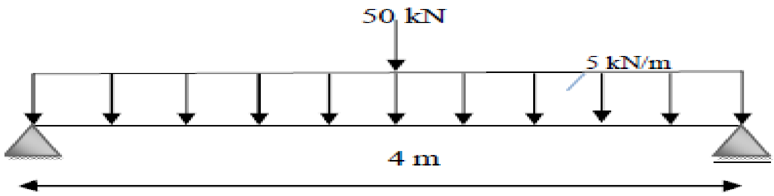


Figure 6. Simply supported beam with 4m span

Table 1: Optimum designs of the castellated beams with 4m span

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO-PSO algorithm	**UB 305×102×25**	**125**	**14**	**56**	**96.04**	Hexagonal
CBO algorithm	UB 305×102×25	125	14	64	96.61	
CSS algorithm	UB 305×102×25	125	14	60	96.45	
CBO-PSO algorithm	UB 305×102×25	243	14	_	98.58	Circular
CBO algorithm	UB 305×102×25	243	14	_	98.70	
CSS algorithm	UB 305×102×25	244	14	_	98.62	

End-filled Castellated beams with hexagonal and circular openings are separately designed by using of three algorithms. The best solutions obtained by these methods are given in Table 1. As it can be seen from the same table, the optimum cost for this case is equal to 96.04$ which is related to the hexagonal beam, and it is obtained by CBO-PSO algorithm. Fig. 7 provides the convergence rates of the best results obtained by these algorithms.

5.2 Castellated beam with 8m span

In the second problem the meta-heuristic algorithms are used to design a simply supported castellated beam with a span of 8m. The beam carries a uniform dead load 0.40 kN/m, which includes its own weight. In addition, it is subjected to two concentrated loads; dead load of 70 kN and live load of 70 kN as shown in Fig. 8. The allowable displacement of the beam is limited to 23 mm. The number of CBs is taken as 50. The maximum number of iterations is considered 200.

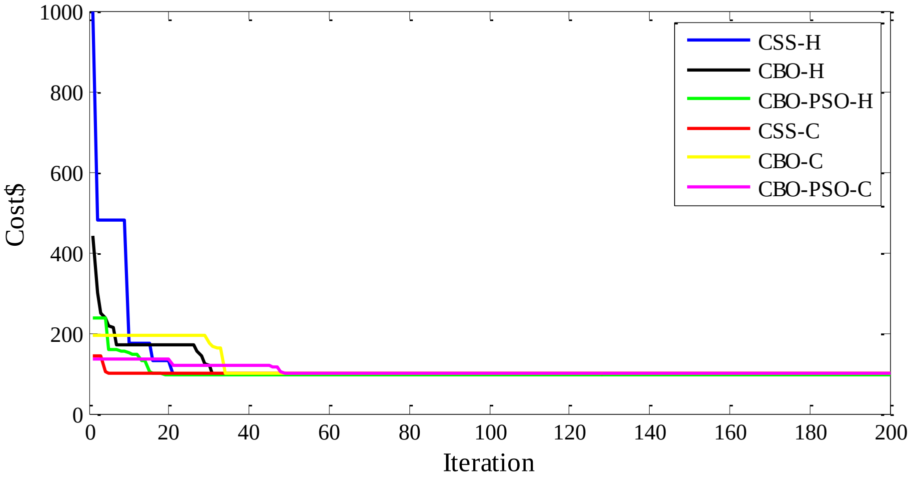


Figure 7. Convergence rates of the best results in design of 4m span beam

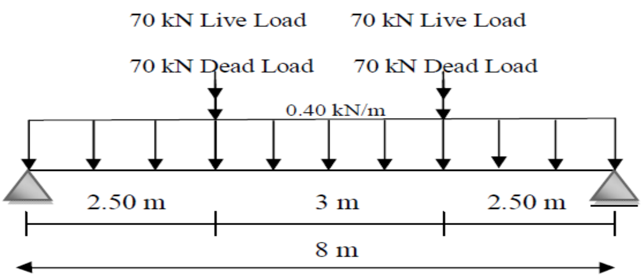


Figure 8. A simply supported beam with 8m span

The beam with 8m span is separately designed by three algorithms. Table 2 compares the results obtained by these methods. As it can be seen, in the optimum design of castellated beam with hexagonal hole, the CBO-PSO approach have been good performance in comparison with other methods, while in the cellular case, both CSS and CBO-PSO led to the same answer. The minimum cost of design is equal to 744.42$, which it is obtained for hexagonal opening. Similar to the previous example, the strength constraints are dominant in the design process. The maximum ratio of these criteria is equal to 0.99 for the Vierendeel mechanism.

Table 2: Optimum designs of the castellated beams with 8m span

Method	Optimum UB section	Hole diameter or cutting depth(mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO-PSO algorithm	**UB 610×229×101**	**246**	**14**	**55**	**744.42**	Hexagonal
CBO algorithm	UB 610×229×101	246	14	58	745.48	
CSS algorithm	UB 610×229×101	246	14	56	744.65	

CBO-PSO algorithm	UB 610×229×101	478	14	_	753.74	
CBO algorithm	UB 610×229×101	479	14	_	754.02	Circular
CSS algorithm	UB 610×229×101	478	14	_	753.74	

The optimum shapes of the hexagonal and circular openings are illustrated separately as shown in Fig. 9.

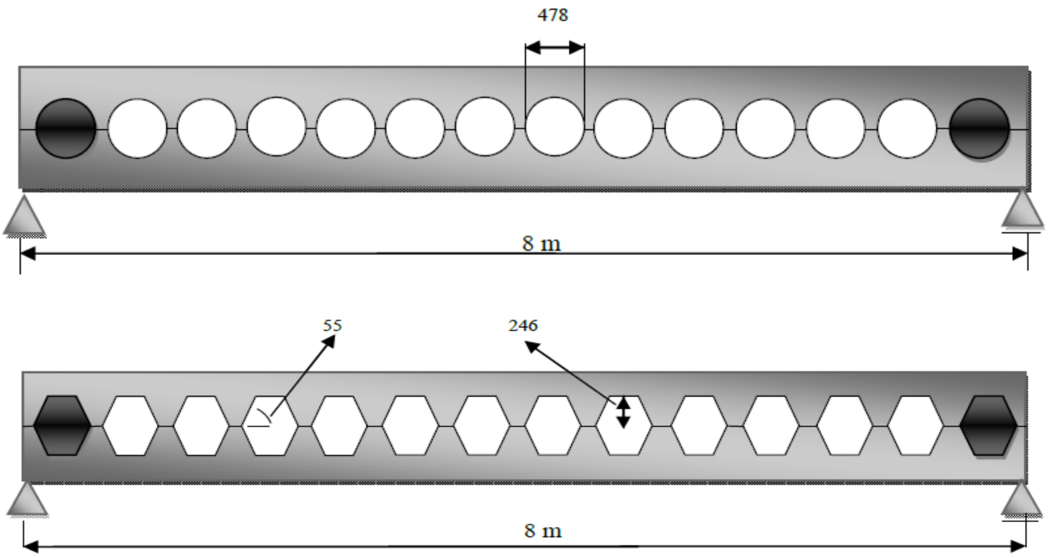


Figure 9. Optimum profiles of the castellated beams with cellular and hexagonal openings

5.3 Castellated beam with 9m span

The beam with 9m span is considered as the last example of this study in order to compare the minimum cost of the castellated beams. The beam caries a uniform load of 40 kN/m including its own weight and two concentrated loads of 50 kN as shown in Fig. 10. The allowable displacement of the beam is limited to 25 mm. Similar to the two previous examples the number of CBs is taken as 50 and the maximum number of iterations is considered 200.

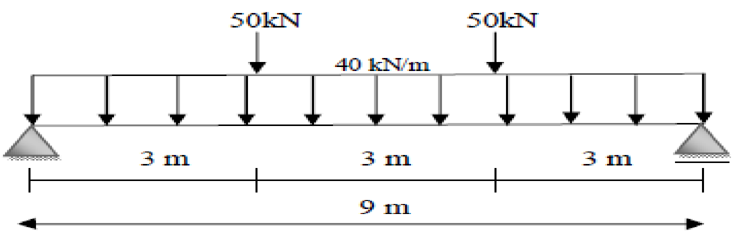


Figure 10. Simply supported beam with 9m span

The optimum results obtained by three meta-heuristic methods are shown in Table 3. In design of castellated beam with hexagonal hole, CBO-PSO algorithm selects 684×254×125 UB profile, 14 holes, and 277 mm for the cutting depth and 58 for the cutting angle. The cost of design is equal to 1031.92$ that it is the lowest in between all of the responses. Also, in the optimum design of cellular beam, all algorithms selects 684×254×125 UB profile, 14 holes and 539 mm for the holes diameter. As it can be seen from Table 3, all three methods have almost the same performance in this design problem.

Table 3: Optimum designs of the castellated beams with 9m span

Method	Optimum UB section	Hole diameter or cutting depth(mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO-PSO algorithm	**UB 684×254×125**	**277**	**14**	**58**	**1031.92**	Hexagonal
CBO algorithm	UB 684×254×125	277	14	60	1034.07	
CSS algorithm	UB 684×254×125	277	14	61	1033.32	
CBO-PSO algorithm	UB 684×254×125	539	14	_	1041.68	Circular
CBO algorithm	UB 684×254×125	539	14	_	1041.79	
CSS algorithm	UB 684×254×125	539	14	_	1041.71	

Fig. 11 shows the convergence of the algorithms in design of the end-filled beam with 9-m span.

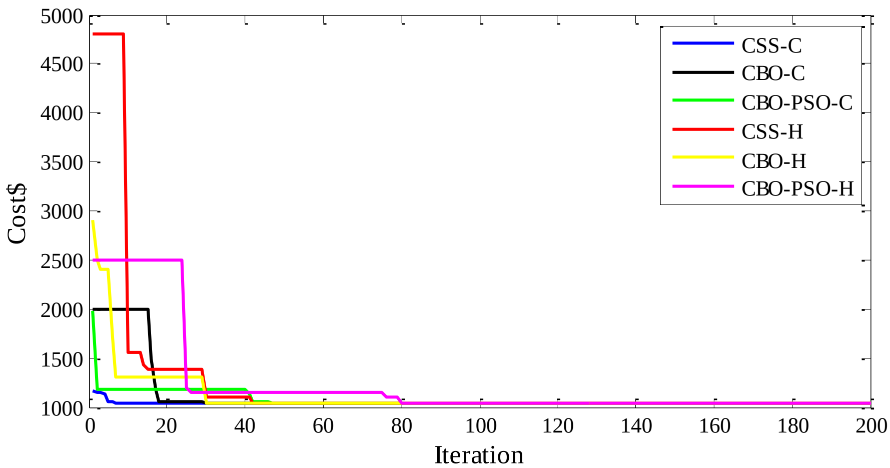


Figure 11. The convergence rate of the algorithms in design of the beam with 9-m span

6. CONCLUDING REMARKS

In this paper, three population-based meta-heuristic algorithms are used in order to design of end-filled castellated beams. In these beams, it is assumed that the end holes of the beam have been filled with steel plates. Thus, the cost of plates is added to the final cost. Three samples are selected from literature to design by these methods. Beams with hexagonal and circular openings are considered as web-opening of castellated beams. Also, the cost of the beam is considered as the objective function. A comparison of the optimal solution is performed between three methods. It is observed that the optimization results obtained from CBO-PSO algorithm for most of the design examples have less cost in comparison to the results of the other algorithms. Likewise, from the results obtained in this study, it can be concluded that the use of beam with hexagonal opening leads to the use of less steel material and it is better than cellular beam from the cost point of view.

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