STRUCTURAL DAMAGE PROGNOSIS BY EVALUATING MODAL DATA ORTHOGONALITY USING CHAOTIC IMPERIALIST COMPETITIVE ALGORITHM

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ABSTRACT

Presenting structural damage detection problem as an inverse model-updating approach is one of the well-known methods which can reach to informative features of damages. This paper proposes a model-based method for fault prognosis in engineering structures. A new damage-sensitive cost function is suggested by employing the main concepts of the Modal Assurance Criterion (MAC) on the first several modes' data. Then, Chaotic Imperialist Competitive Algorithm (CICA), a modified version of the original Imperialist Competitive Algorithm (ICA) which has recently been developed for optimal design of complex trusses, is employed for solving the suggested cost function. Finally, the optimal solution of the problem is reported as damage detection results. The efficiency of the proposed method for damage identification is evaluated by studying three numerical examples of structures. Several single and multiple damage patterns are simulated and different number of modal data are utilized as input data (in noise free and noisy states) for damage detection via suggested method. Moreover, different comparative studies are carried out for evaluating the preference of the suggested method. All the obtained results emphasize the high level of accuracy of the suggested method and introduce it as a viable method for identifying not only damage locations, but also damage severities.

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1. INTRODUCTION

Structural Health Monitoring (SHM) is a multi-step program aimed at evaluating structural condition by estimating location and severity of existed damages. Early detection of damages, not only can prevent some calamitous events, but also can help engineers to prepare appropriate plans for structural rehabilitation. Damage detection problem can be defined as a highly ill-posed inverse problem, which considers gathered vibrational properties from field experiments (such as natural frequencies and mode shape vectors) as input data, and try to estimate structural physical properties (as damage features) in a way that the recorded vibrational characteristics can be generated with high level of accuracy [1, 2]. Since optimization algorithms are devoted to finding optimal solutions of an inverse problem with an iterative scheme, it seems more rational that the damage detection problem is defined as an optimization problem. In this regards, different studies can be found in the literature which have utilized optimization-based strategies for damage quantification as well as damage localization [3–14].

Lee et al. [3] used topology optimization for damage identification in planar structures by considering some points of the frequency response functions of the damaged structure as controlling points. Bagheri et al. [6] employed free vibration equilibrium to formulate inverse problem of damage identification and utilized original version of imperialist competitive optimization algorithm for finding optimal solution of the problem. Moradi and Kargozarfard [13] proposed a method for multiple crack identification in beam structures by employing bee optimization algorithm for solving an objective function which was formulated by considering changes of the eigen-frequencies and some strain energy parameters. Mohan et al. [8] investigated the applicability of the genetic algorithm and particle swarm optimization for crack identification with a cost function which inspected changes in the natural frequencies. Ghodrati Amiri et al. [14] proposed a model updating approach by means of generalized flexibility matrix and democratic particle swarm optimization. They assessed the applicability of their method by studying different numerical examples of engineering structures. Li and Lu [15] employed multi-swarm fruit fly optimization algorithm for solving a cost function which was based on the differences of the natural frequencies and mode shapes of damaged and intact state of a structure. To detect faults in trusses, Kaveh and Mahdavi [12] suggested an optimization-based methodology and illustrated that the enhanced colliding bodies optimization algorithm performs better than colliding bodies optimization in solving the inverse problem of structural damage detection. By means of calculated static deflections from modal data and cuckoo optimization algorithm, Zare Hosseinzadeh et al. [16] identified structural damages in the engineering structures. They verified their method by different numerical and experimental studies.

By perusing above mentioned researches, it can be concluded that there is a very strong relation between feeding precise input data and getting accurate results about structural damages. This fact has a vital importance in planning for solving inverse problems, especially in the presence of highly ill-posed conditions. It should be mentioned that although the inverse problems are naturally ill-posed, this condition may be intensified in some cases. In the field of structural damage detection, noise effects, complex structural models and modelling errors are some of the main sources which can cause a highly ill-

posed situation. To propose an effective damage detection approach which can perform well in the ill-posed and complex solution domains, two important conditions should be met:

- The proposed objective function not only should be damage-sensitive, but also should declare a one-to-one relation between suggested damage index and each special arrangement of structural physical properties. In addition, the objective function should be formed by means of the least possible input data to decrease unavoidable effects of uncertainties in the input data (such as noise).
- The optimization approach should follow a powerful strategy to search the solution domain in a random but rational way, to prevent from trapping by local extremums.

This paper is aimed at presenting a novel model updating approach for structural damage identification by considering all above mentioned challenges. In this method, only the first several modes' data (i.e. natural frequencies and mode shape vectors) are considered as input data. Then, Modal Assurance Criterion (MAC) is employed to inspect the orthogonality conditions between mode shape vectors of the damaged and analytical models. Moreover, the approaching of the natural frequencies of the damaged and analytical models is evaluated by adding a simple mathematical concept to the proposed MAC-based cost function. Finally, the Chaotic Imperialist Competitive Algorithm (CICA) is employed for solving the problem and finding structural damages. CICA is a modified version of the original ICA which is proposed by Talatahari et al. [17] for optimal design of trusses. The performance of the proposed damage detection method is demonstrated by studying different damage patterns on three numerical examples of structures, namely a planar steel truss, a two-span continuous concrete beam and a space steel frame. In addition, different comparative studies are carried out to evaluate the robustness of the CICA in comparison with other evolutionary optimization approaches.

The paper is organized as follows. The overview of the CICA is presented in Section 2. It is followed by Section 3 which describes the details of the proposed damage identification method. The numerical studies are presented in Section 4 and finally, conclusion remarks are presented in section 5.

2. CHAOTIC IMPERIALIST COMPETITIVE ALGORITHM (CICA)

The original Imperialist Competitive Algorithm (ICA) is a global search optimization method that is inspired from a socio-political competitive event [18]. The aim of the algorithm is to find a global extremum the argument $\mathbf{Y}$ of a given function $f(\mathbf{Y})$. Similar to other evolutionary optimization algorithms, this algorithm starts with an initial population that is called *country*. Each country is presented as a vector:

$$\mathbf{country} = \left\{ y_1 \quad y_2 \quad ... \quad y_{N_v} \right\}^T \tag{1}$$

where y_n is the *n*-th variable, and N_v represents the number of variables in the function. The cost of each country is calculated as:

$$c = f(\mathbf{country}) \tag{2}$$

Some of the best countries (with low cost) are selected as the *imperialist*, which are the initial candidates for the optimal solution. The rest of the initial population is considered as *colony* and are divided among the imperialists. It should be mentioned that the division of colonies is directly proportional to the power of every empire. After the initialization process, the imperialistic countries begin to improve their colonies and attempt to absorb new colonies. This is called as the *assimilation process* which is modeled by moving all of the colonies toward the imperialist along different optimization axes. To ensure that many positions are explored in search of the minimal cost, the assimilation of the colonies by the imperialists does not occur through the direct movement of the colonies toward the imperialist. A random path is induced by a random amount of deviation added to the direction of the movement [18]. This step can be represented as below:

$$\mathbf{X}_{new} = \mathbf{X}_{old} + U\left(0, \beta \times d\right) \times \mathbf{V}_1 \tag{3}$$

in which $\mathbf{X}_{old}$ and $\mathbf{X}_{new}$ are the current and new positions of the colony, respectively. Moreover, β and d are the control parameter and the distance between colony and imperialist, respectively. $U(0, \beta \times d)$ denotes random value which is uniformly distributed between 0 and $\beta \times d$. Also, $\mathbf{V}_1$ is a unit vector and its start point is the current location of the colony and its direction is toward the imperialist location.

If during the assimilation process, a colony reaches a position with lower cost than the imperialist, the imperialist and the colony switch their positions. Then, the algorithm will continue with the imperialist in the new position and the colonies will be assimilated by the new arrangement of imperialists.

All empires try to take the possession of colonies of other empires and control them. This competition is modeled by just picking some of the weakest colonies of the weakest empire and making a competition among all empires to possess these colonies. The described process is repeated again from the assimilation step, and empires with no colony are eliminated in the process. Finally, the optimization algorithm is stopped when one empire only is left or the number of iterations reaches to the defined maximum level. More details of original ICA can be found at [18, 19].

Despite of acceptable performance of the original ICA in solving inverse problems, for providing a situation in which a large number of candidate solutions are randomly selected and evaluated in searching highly complex solution domains, Kaveh and Talatahari [20] proposed orthogonal ICA by modifying assimilation process using not only different random values, but also by considering orthogonal colony-imperialistic contacting line to add some rational deviation in locating final position of the colony in its movements toward the imperialists. Fig. 1 shows assimilation process for original and orthogonal ICA in a two-dimensional problem. For orthogonal ICA, the assimilation process is defined as below [20, 21]:

$$\mathbf{X}_{new} = \mathbf{X}_{old} + \left(\beta \times d \times \mathbf{rand} \otimes \mathbf{V}_1\right) + \left(U\left(-1, +1\right) \times \tan(\theta) \times d \times \mathbf{V}_2\right) \tag{4}$$

where, $\mathbf{V}_2$ is a unit vector which is perpendicular to $\mathbf{V}_1$. $\mathbf{rand}$ is a vector of random numbers and $\otimes$ denotes element-by-element multiplication. By this modification, ICA promoted to

an evolutionary optimization approach which could efficiently evaluate more points in the solution domain of complex problems. For providing a situation in which the solution domain is sought by a fast speed strategy, Talatahari et al. [17] employed chaos theory and proposed Chaotic Imperialist Competitive Algorithm (CICA) which promoted the speed of orthogonal ICA. Generally, those optimization algorithms which are based on the chaos theory can be considered as stochastic search methodologies which differ from any of the existing evolutionary computation and swarm intelligence methods. Because of the non-repetition of chaos, chaotic optimization algorithms can search the complex solution domains with fast speed in comparison with probability-based methods [17]. The assimilation process for CICA is introduced as:

$$\mathbf{X}_{new} = \mathbf{X}_{old} + \left(\beta \times d \times \mathbf{CM} \otimes \mathbf{V}_1\right) + \left(CM \times \tan(\theta) \times d \times \mathbf{V}_2\right) \tag{5}$$

where, *CM* is the chaotic variable (generated based on the chaotic map) which is used instead of random numbers. Readers can find more information about CICA in [17].
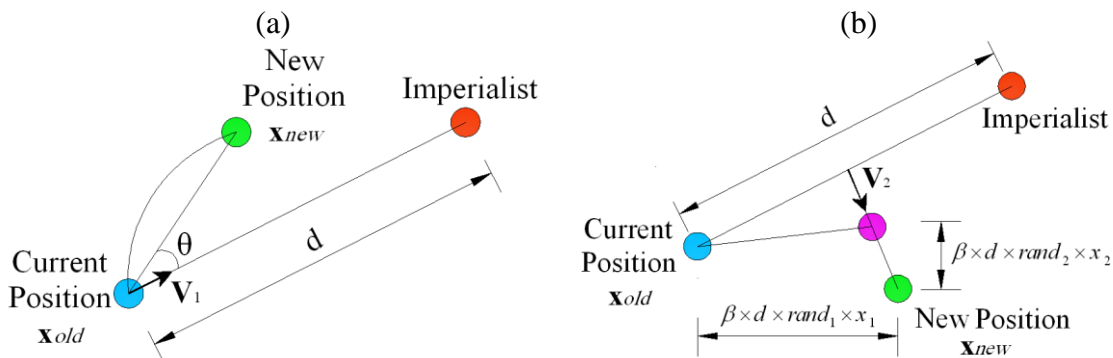

Figure 1. Assimilation process for (a) original Imperialist Competitive Algorithm (ICA) proposed by Atashpaz-Gargari and Lucas [18], and (b) orthogonal ICA proposed by Kaveh and Talatahari [20]

3. PROPOSED METHOD

This section is devoted to describing the details of the suggested method. Since the proposed method is a model-based approach, in the first step a numerical model of the damaged structure (with unknown damage severity) should be constructed. This is done by considering an appropriate definition of damage in the analytical finite element model of the monitored structure. In this paper, damage is defined as some reduction in the stiffness matrix of the damaged elements. Therefore, the global stiffness matrix for the analytical model of damaged structure ($\mathbf{K}^a$) can be formulated as:

$$\mathbf{K}^a = \bigcup_{i=1}^{N_e} (1-\alpha_i)\mathbf{k}_i^u \ , \ 0 \le \alpha_i \le 1.0 \tag{6}$$

in which, $\mathbf{k}_i^u$ and α_i are the stiffness matrix of the i-th element in the undamaged state and damage severity of the i-th element, respectively. Also, N_e is the number of elements on the finite element model of the monitored structure. For the analytical model, if a special arrangement of damage severities is assumed, the modal data can be extracted via classic modal analysis using below mentioned equilibrium:

$$\left(-\left(\omega_i^2\right)_a \mathbf{M} + \mathbf{K}^a\right)\left(\boldsymbol{\varphi}_i\right)_a = \mathbf{0} \ , \ i = 1,2,3,...,N \tag{7}$$

where $\mathbf{M}$ is the mass matrix, ω_i and $\boldsymbol{\varphi}_i$ are the i-th natural frequency and mass-normalized mode shape vector related to the i-th mode, respectively. In addition, a is a sign that indicate the presented parameter is related to the analytical model of the monitored structure and N is the total number of degrees of freedom (DOFs).

By having access to the first p modes' natural frequencies and related mass-normalized mode shape vectors of the analytical model (generated from modal analysis based on each arrangement of damage severities) and monitored structure (extracted from recorded data from field experiments), the proposed cost function can be introduced. This cost function is based on two concepts. First one is related to measuring amount of orthogonality of mode shape vectors by means of Modal Assurance Criterion (MAC). Generally, MAC is a geometrical index which can measure amount of correlation between two vectors [22]. Here, we employ MAC for measuring amount of correlation between mode shape vectors of the monitored and numerical models. For instance, MAC is defined for vectors $(\boldsymbol{\varphi}_i)_a$ (i-th mode shape of the analytical model) and $(\boldsymbol{\varphi}_j)_d$ (j-th mode shape of the monitored structure) as:

$$MAC\left(\left(\boldsymbol{\varphi}_i\right)_a, \left(\boldsymbol{\varphi}_j\right)_d\right) = \frac{\left|\left(\boldsymbol{\varphi}_i\right)_a^T \cdot \left(\boldsymbol{\varphi}_j\right)_d\right|^2}{\left\{\left(\boldsymbol{\varphi}_i\right)_a^T \cdot \left(\boldsymbol{\varphi}_i\right)_a\right\} \cdot \left\{\left(\boldsymbol{\varphi}_j\right)_d^T \cdot \left(\boldsymbol{\varphi}_j\right)_d\right\}} \tag{8}$$

If MAC is equal to 1.0, a complete accordance between two vectors can be reported. By considering this fact, we can claim that the analytical model behaves so closely to the monitored structure if the below mentioned equation is met:

$$MAC\left(\left(\boldsymbol{\varphi}_i\right)_a, \left(\boldsymbol{\varphi}_j\right)_d\right) = 1.0 \ , \ i = j \quad , \quad i,j = 1,2,3,...,p \tag{9}$$

It is worth noting that because of orthogonality, the below mentioned equation is honest when the analytical model has a complete similarity with the monitored structure:

$$MAC\left(\left(\boldsymbol{\varphi}_i\right)_a, \left(\boldsymbol{\varphi}_j\right)_d\right) = 0.0 \ , \ i \neq j \quad , \quad i,j = 1,2,3,...,p \tag{10}$$

However, based on the main definition of the MAC [22] and because of existing some unavoidable uncertainties in the recorded data from monitored structure, it is possible that in some cases Eq. (10) is satisfied with values lower than 0.4 and close to 0 (not exactly equal

to zero). Therefore, Eq. (9) is considered in this paper for checking orthogonality property.

The second concept that we use to suggest a new damage-sensitive cost function is about a mathematical issue which makes it possible to evaluate the accordance of the first p modes' natural frequencies between analytical and monitored models by only a single cost function. This is done by employing the main strategy introduced with Eq. (9). **DMAC** is defined as below:

$$\mathbf{DMAC} = \left\{ MAC\left((\varphi_1)_a, (\varphi_1)_d\right) \quad MAC\left((\varphi_2)_a, (\varphi_2)_d\right) \quad ... \quad MAC\left((\varphi_p)_a, (\varphi_p)_d\right) \right\}^T \tag{11}$$

If the vectors of the first p modes' natural frequencies for the monitored and analytical models are shown with $\mathbf{\Omega}^d$ and $\mathbf{\Omega}^a$, respectively, the proposed objective function can be introduced as below:

$$C(\alpha_1, \alpha_2, ..., \alpha_{N_e}) = \left\|\left(\mathbf{\Omega}^d \otimes \mathbf{DMAC}\right) - \mathbf{\Omega}^a\right\|^2 \tag{12}$$

where $\| \ \|$ is the Euclidean length. Also, $\otimes$ denotes element-by-element multiplication. At the next step, the CICA, which is described in the previous section, is employed for solving Eq. (12) to find global extremums as damage detection results. As it is mentioned before, the CICA is a modified version of the original ICA which is proposed by Talatahari et al. [17] for optimal design of complex truss structures.

4. NUMERICAL STUDIES

4.1 Planar steel truss

As the first example, the presented method is employed for damage localization and quantification in the planar steel truss which has been introduced by Ghodrati Amiri et al. [14]. The finite element model of this truss consists of 29 elements, with two DOFs at each free node (see Fig. 2). Its material properties are as follows: modules of elasticity $E=200$ GPa, mass density $\rho=7850$ kg/m^3, the mass per unit length and cross sectional area for vertical members are $m=39.25$ kg/m and $A=0.005$ m^2, and those for bottom horizontal members are $m=3000$ kg/m and $A=0.010$ m^2, and those for top horizontal members are $m=78.50$ kg/m and $A=0.010$ m^2, and those for the diagonal members are $m=62.80$ kg/m and $A=0.008$ m^2, respectively.
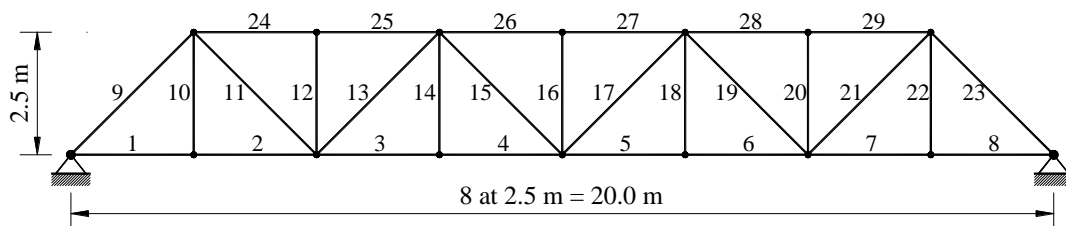

Figure 2. Finite element model of planar steel truss

Table 1 describes details of the studies three damage patterns. The first pattern simulates a single damage case; however, the second and third patterns present multiple damage cases with different damage severities. Since in real cases it is possible that the input data are polluted with different levels of random noises, in the present study not only noise free state, but also noisy state is considered. To simulate noise in the natural frequencies and mode shape vectors, we follow below mentioned strategies:

$$\omega_i^n = \omega_i \left(1 + \kappa \theta_i\right) \tag{13a}$$

$$\boldsymbol{\varphi}_i^n = \boldsymbol{\varphi}_i \left(1 + \varepsilon \boldsymbol{\eta}_i\right) \tag{13b}$$

Table 1: Details of the simulated damage patterns in the planar steel truss

Pattern Number	Scenario Explanation	
	Element	Damage (%)
1	17	15
2	4, and 20	10, and 15
3	9, 19, and 26	20, 30, and 20

in which ω_i^n and $\boldsymbol{\varphi}_i^n$ are the i-th natural frequency and mode shape vector, which are contaminated with noise, κ and ε are the noise levels, and θ_i and $\boldsymbol{\eta}_i$ are a random value and random number vector, respectively. The random numbers are values between [-1 1] which are generated with MATLAB software. In this example, it is assumed that κ and $\varepsilon=0\%$ for free noise state. Moreover, for noisy state, two different cases are considered: Noise I: $\kappa=1.5\%$, $\varepsilon=0.5\%$; and Noise II: $\kappa=3\%$, $\varepsilon=1\%$. In the present example, the number of utilized modal data (p) is considered as 6 and 9, to investigate the performance of the method when different numbers of modal data are employed. The selected parameters for CICA are as follows: number of countries = 100; number of imperialists = 10; maximum number of iterations = 1000; $\beta = 2.0$; $\tan(\theta) = 1.0$, and CM = sinusoidal map: $x_{k+1} = \sin(\pi x_k)$. It should be mentioned that these parameters are selected by trial and error, based on the suggested recommendation in [17].

After generating input data for each case of the presented damage patterns, the proposed method is employed to solve the damage detection problem. Figs. 3-5 show the obtained results for the simulated damage patterns. As it can be seen, the presented method is able to localize and quantify damages with high level of accuracy whether the ideal (free noise) input data are fed or noisy ones. In addition, about the effects of p, it can be concluded that the effects of noise tenses if the number of utilized modes increases. It is worth noting that in the noisy states it is possible that some damage severities are reported for healthy elements, but, this issue cannot weaken the robustness of the method, because of small severities for such reported damages. This fact that all results have been reported with an error less than 10% in all cases is strong evidence for the former claim.

To numerically investigate the robustness of the CICA in solving presented problem, the third damage pattern (when $p=6$ and the input data are contaminated with Noise II) is solved again by three other optimization algorithms, named: original ICA [18], Big Bang-Big Crunch (BB-BC) [23], and Genetic Algorithm (GA) [24]. For all of these optimization approaches, the maximum number of iterations is considered 1000. Also, other optimization

parameters are selected in a way that the problem for noise free state could be solved with a level of accuracy more than 98%. It should be mentioned that to provide an acceptable situation for comparing these algorithms with CICA, the input data are the same as those which were fed in the above mentioned investigations. Moreover, it is worth noting that all these algorithms are repeated for ten times and only the results of the best run are considered as damage detection results. The obtained results are summarized in Table 2, for CICA, ICA, BB-BC, and GA. In this table, for comparing results with more convenience, not only are the false results denoted with '*', but also the comparative errors for damaged elements are calculated based on Eq. (14) and presented in parentheses:

$$Comparative Error\ (\%) = \frac{Estimated\ Damage - Actual\ Damage}{Actual\ Damage} \times 100 \tag{14}$$

From this table, it can be concluded that although original ICA has reported results close to the CICA, the BB-BC and GA have revealed some mistakes in detecting damage locations and/or extents. To evaluate the convergence speed of these optimization algorithms, the convergence curves for all of them (in the third damage pattern with p=6 and Noise II) are shown in Fig. 6. As it is obvious, the CICA and original ICA converge in the ~500th and ~620th iteration, respectively. However, BB-BC and GA cannot converge to the global extremums in the 1000 iterations and it is possible that they can find optimal solution with more iterations.
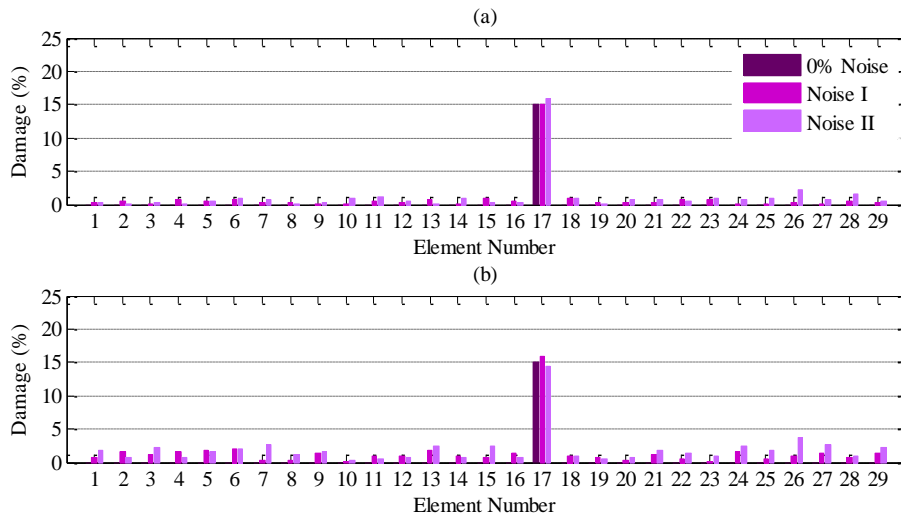

Figure 3. Damage identification results for damage pattern 1 in the planar steel truss using: (a) the first six modes' data, and (b) the first nine modes' data
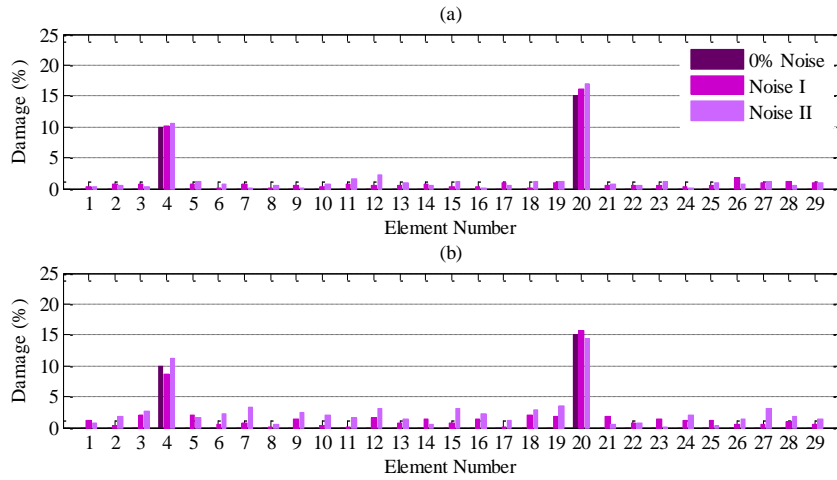
Figure 4. Damage identification results for damage pattern 2 in the planar steel truss using: (a) the first six modes' data, and (b) the first nine modes' data
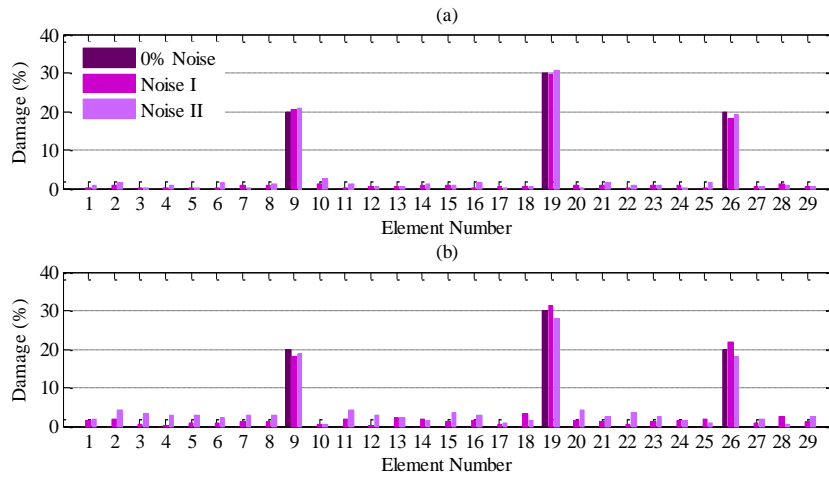

Figure 5. Damage identification results for damage pattern 3 in the planar steel truss using: (a) the first six modes' data, and (b) the first nine modes' data

Table 2: Obtained results for damage pattern 3 in the planar steel truss using CICA, ICA, BB-BC, and GA (the input data is the first six modes' data pulloted with Noise II)

Element Number	Damage Severity (%)				
	Actual	CICA	ICA	BB-BC	GA
1	0	0.86	0.34	1.19	1.00
2	0	1.59	1.30	1.04	1.52
3	0	0.13	2.96	0.13	0.65
4	0	0.71	1.30	0.34	0.76
5	0	0.17	0.90	2.44	1.60
6	0	1.54	1.09	1.88	0.69
7	0	0.01	0.59	0.18	2.14

8	0	1.24	1.49	11.94*	2.55
9	20	20.95 **(4.75%)**	22.38 **(11.89%)**	12.14 **(-39.31%)**	14.90 **(-25.51%)**
10	0	2.39	1.37	0.85	0.89
11	0	1.14	0.37	0.64	1.52
12	0	0.64	0.74	1.01	0.28
13	0	0.42	1.25	0.93	2.36
14	0	1.28	1.56	1.17	6.69*
15	0	0.69	0.16	1.16	2.13
16	0	1.46	1.86	0.43	7.68*
17	0	0.29	1.55	0.91	1.55
18	0	0.42	0.97	0.26	0.06
19	30	30.72 **(2.41%)**	30.85 **(2.84%)**	32.50 **(8.35%)**	25.11 **(-16.31%)**
20	0	0.22	0.89	0.97	0.81
21	0	1.39	0.61	5.65*	0.42
22	0	0.93	1.02	0.62	0.46
23	0	0.88	1.02	1.18	1.10
24	0	0.23	1.64	0.79	0.24
25	0	1.36	1.59	0.80	1.56
26	20	19.00 **(-5.02%)**	19.29 **(-3.56%)**	23.12 **(15.59%)**	9.22 **(-53.88%)**
27	0	0.56	6.16*	1.05	1.81
28	0	0.82	1.62	0.75	4.82*
29	0	0.64	1.07	0.24	1.66

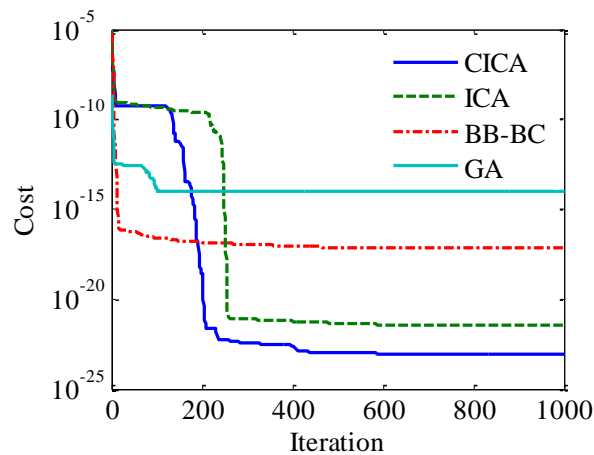( ): denotes comparative errors; *: denotes false results

Figure 6. The convergence curves for the damage pattern 3 of the planar steel truss (when the first six modes' data (with Noise II) are utilized) for CICA, ICA, BB-BC, and GA

This section ends with another study which is concentrated on evaluating the orthogonality property between updated model (based on the results of a typical iteration in all above mentioned optimization approaches) and monitored structure. For this purpose, the

updated model based on the results of the 600th iteration is considered as analytical model, then, the orthogonality property is controlled for all six modes by calculating below mentioned equation:

$$MAC(i,j) = MAC\left(\left(\varphi_i\right)_a, \left(\varphi_j\right)_d\right) , \ i,j = 1,2,...,6 \tag{15}$$

Fig. 7 shows a space view of the calculated MACs (based on Eq. (15)) for the results related to the best run of each optimization algorithm. As obvious, although the related counter to the CICA has an acceptable concurrence with Eqs. (9) and (10), the others show some deviations from these equations (especially from Eq. (9), as the main orthogonality property which considered in the present paper) and this means that the CICA can converge to the optimal solution with fast speed and high accuracy. Based on these interpretations, it can be concluded that the CICA is able to search the solution domain with a considerable accuracy and there is low risk that it is trapped with local extremums.
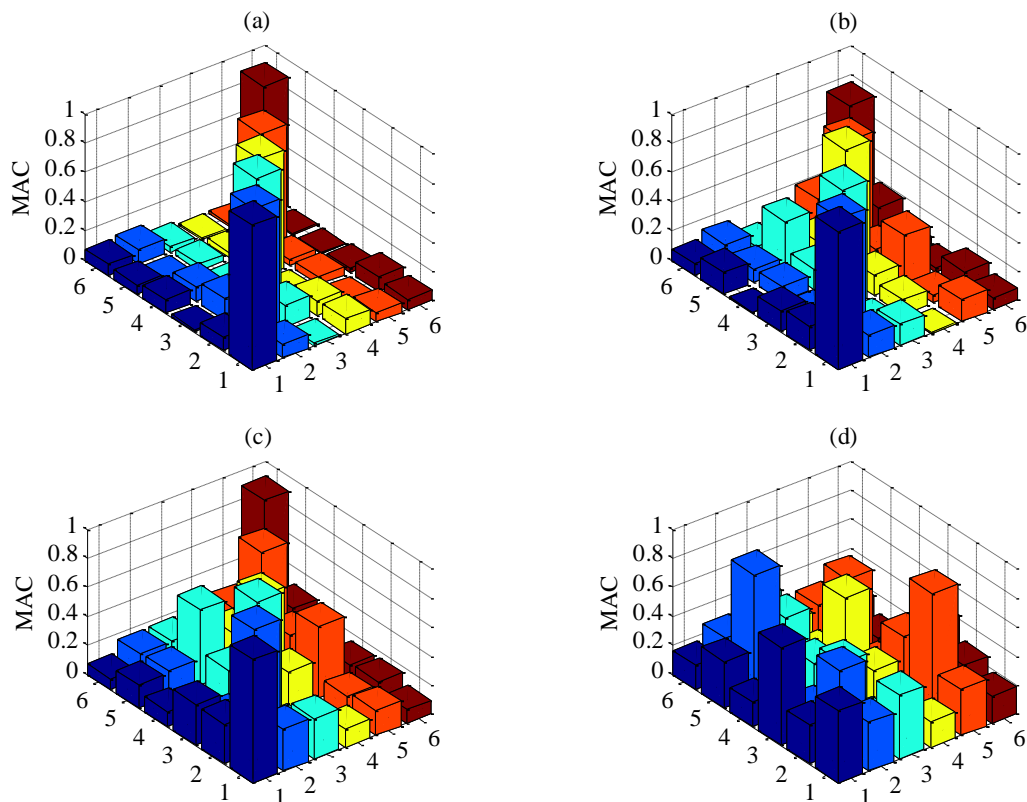

Figure 7. Calculated MAC values for the first six mode shape vectors in the 600th iteration of the optimization in solving the third damage pattern of the planar steel truss (for *p*=6, and Noise II) using: (a) CICA, (b) ICA, (c) BB-BC, and (d) GA

4.2 Two-span continuous concrete beam
The second example is devoted to damage detection in a continuous concrete beam with 18

elements and 19 nodes (with two DOFs at each node). Fig. 8 shows the finite element model of this beam. Its material properties are as follows: The Young's modulus and mass density of beam are considered as $E=25$ *GPa* and $\rho=2500$ *kg/m^3*, respectively. The cross sectional area and the moment of inertia for all elements are equal to $A=0.45$ *m^2* and $I=0.01832$ *m^4*, respectively.
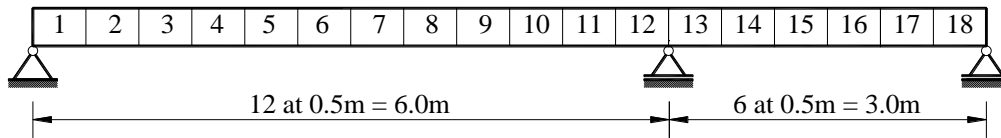

Figure 8. Finite element model of the two-span continuous concrete beam

In this example, two multiple damage cases which are described in Table 3 are simulated. The problem is solved for two different numbers of available modal data ($p=4$ and $p=6$) in noise free and noisy states (noisy states are: Noise I and Noise II, which are described in the pervious example). After generating input data, the proposed cost function is constructed. Then, the CICA is performed for finding optimal solution of the problem. All parameters of the CICA are similar to the previous example. The obtained results are shown in Figs. 9 and 10, for damage pattern 1 and 2, respectively. It can be seen that for both patterns, the results are acceptable. Similar to the previous example, some differences between simulated and detected damages may be reported; however, the amounts of errors in such cases are small and cannot inversely affect the robustness of the presented method.

Table 3: Details of the simulated damage patterns in the two-span continuous concrete beam

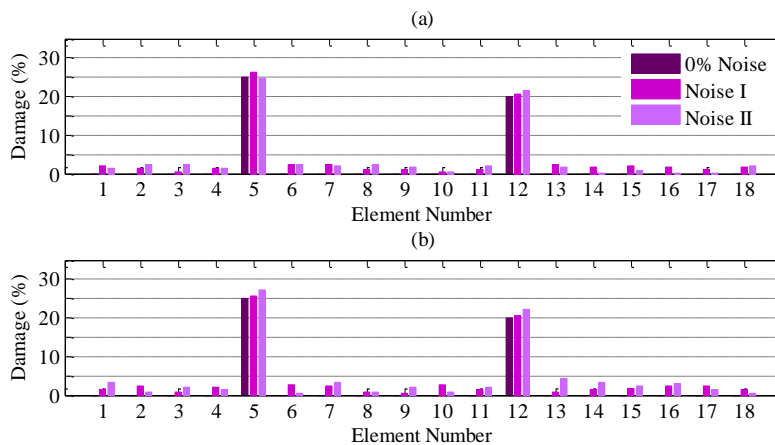| Pattern Number	Scenario Explanation	
	Element	Damage (%)
1	5, and 12	25, and 20
2	4, 9, 14, and 17	15, 15, 25, and 30

Figure 9. Damage identification results for damage pattern 1 in the two-span continuous concrete beam using: (a) the first four modes' data, and (b) the first six modes' data
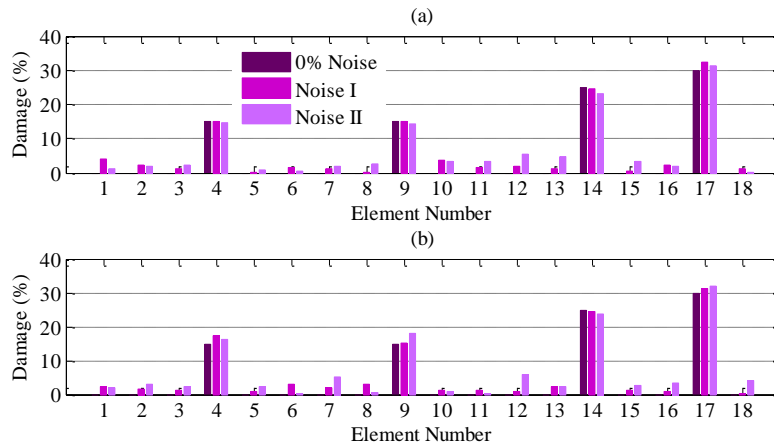
Figure 10. Damage identification results for damage pattern 2 in the two-span continuous concrete beam using: (a) the first four modes' data, and (b) the first six modes' data

In the previous section, the fast convergence of the CICA is revealed by showing the convergence curves. In this section, for investigation the speed and robustness of the CICA in solving the optimization problem, the obtained damage severities at different steps of the optimization procedure are evaluated for the first damage case in the two-span continuous concrete beam, when $p=4$ and the modal data are polluted with Noise I. Fig. 11(a) shows damage extents for the two damaged elements in all of the 1000 iterations. Based on these figure, CICA can find expected damage severity after ~200 iterations in both damaged elements (see Fig. 11(b)). Therefore, the fast speed convergence of this optimization algorithm can be concluded.
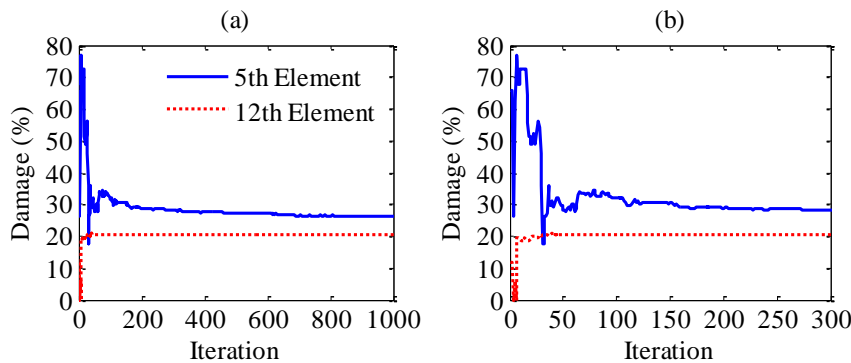

Figure 11. Obtained damage detection results in solving the first damage pattern of the two-span concrete beam ($p=4$ and Noise I) in: (a) all 1000 iterations, and (b) the first 300 iterations

4.3 Space steel frame

In the previous sections, different studies were carried out not only for investigating the applicability of the proposed method for damage identification in planar structures, but also for justifying the necessity of presenting such strategy. In this section, for evaluating the

applicability of the suggested method in health monitoring of complex structures, a space steel frame is studied. The finite element model of this structure consists of 44 elements and each node has six DOFs (see Fig. 12). For this structure, the modules of elasticity and mass density are similar to the first example (E=200 *GPa*, and ρ=7850 *kg/m³*). Moreover, shear modules and cross section of all elements are G=85 *GPa* and 2IPE140, respectively.
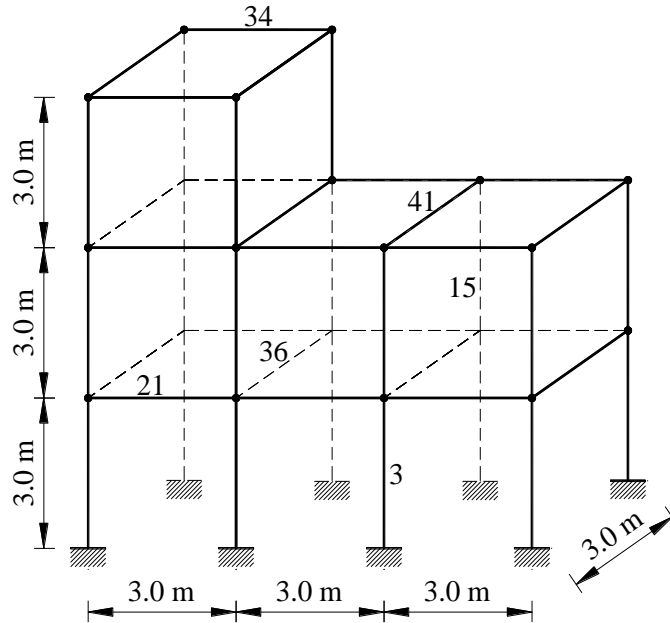

Figure 12. Finite element model of space steel frame

Table 4 shows details of the simulated two damage patterns. The first damage pattern consists of a scenario with two structural damages. However, the second damage pattern simulates a case with four damaged elements. Two different cases are assumed about the number of available modal data: p=10 and p=16. In addition, not only noise free state, but also two noisy states which were studied in the previous examples are considered for contaminating input data with random noises. Selected optimization parameters are as below: number of countries = 200; number of imperialists = 20; maximum number of iterations = 3000; β = 2.0; tan(θ) = 1.0, and *CM* = sinusoidal map: $x_{k+1} = \sin(\pi x_k)$. After construction the cost function, the CICA is employed for solving inverse problem. Figs. 13 and 14 show the obtained results for damage patterns 1 and 2, respectively. By inspecting these figures it can be concluded that the proposed method not only can detect damage locations correctly, but also it can estimate damage severities with an acceptable accuracy in all cases.

Table 4: Details of the simulated damage patterns in the space steel frame

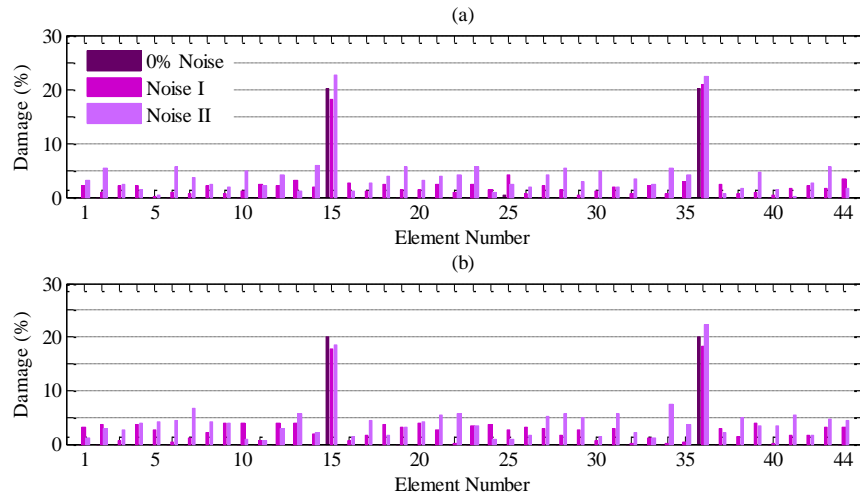| Pattern Number	Scenario Explanation	
	Element	Damage (%)
1	15, and 36	20, and 20
2	3, 21, 34, and 41	20, 30, 25 and 20

Figure 13. Damage identification results for damage pattern 1 in the space steel frame using: (a) the first ten modes' data, and (b) the first sixteen modes' data
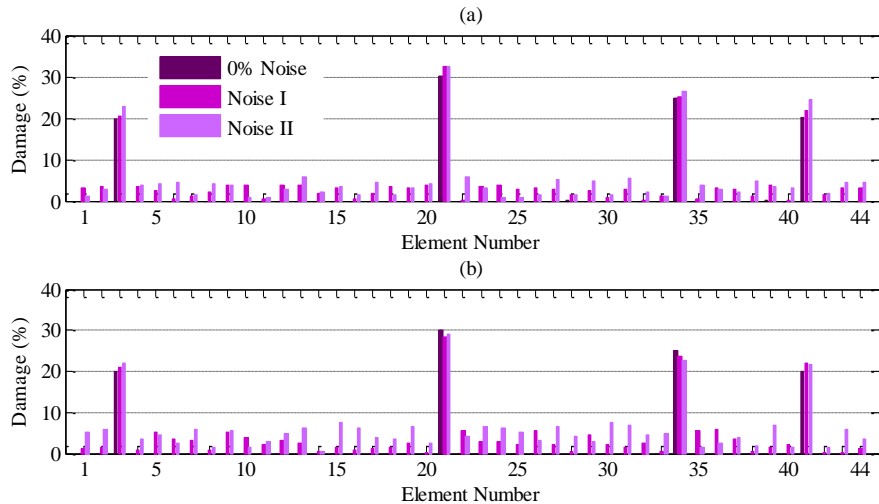

Figure 14. Damage identification results for damage pattern 2 in the space steel frame using: (a) the first ten modes' data, and (b) the first sixteen modes' data

5. CONCLUSIONS

In this paper a new model-based strategy was proposed for damage localization and quantification in engineering structures. First, a damage-sensitive cost function was suggested using the main concepts of the Modal Assurance Criterion (MAC) and its contribution in evaluating the orthogonality of the first several mode shape vectors between monitored and analytical models. Moreover, the accordance of the first several natural frequencies between monitored and analytical models was measured by applying some

mathematical concepts on the same cost function, without adding any other term to the cost function. Therefore, not only can both natural frequencies and mode shape vectors be evaluated for model-updating purposes, but also a robust and effective single objective function can be introduced for formulating damage detection problem as an inverse problem. Finally, the cost function was solved by Chaotic Imperialist Competitive Algorithm (CICA), a modified version of the original ICA which has been developed by Talatahari et al. [17] for optimal design of trusses. The applicability of the presented method was demonstrated by studying different damage patterns on three numerical examples of planar and space structures. In addition, different comparative studies were carried out not only to evaluate the viable performance of the suggested cost function, but also to justify the necessity of using CICA in comparison with other optimization algorithms. The obtained results introduced the proposed method as a powerful strategy for damage identification in engineering structures when the highly ill-posed conditions exist due to limited noisy input data.

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