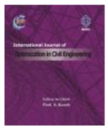
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2018; 8(3):347-355



OPTIMAL SHAKEDOWN ANALYSIS OF LATERALLY LOADED PILE WITH LIMITED RESIDUAL STRAIN ENERGY

M. Movahedi Rad*, †
Department of Structural and Geotechnical Engineering, Széchenyi István University
Egyetem Tér1, H-9026 Győr, Hungary

ABSTRACT

For application of the plastic analysis and design methods the control of the plastic behaviour of the structures is an important requirement. In this study, the complementary strain energy of the residual forces is considered as an overall measure of the plastic performance of the structure. Shakedown theorem for the analysis of the plastic behaviour of the laterally loaded piles is developed and applied to single vertical long pile. Limit curves are presented for the shakedown load multipliers. The formulations of the problems lead to mathematical programming which are solved by the use of nonlinear algorithm.

Keywords: shakedown analysis; complementary strain energy; residual forces.

Received: 12 August 2017; Accepted: 10 October 2017

1. INTRODUCTION

The most important tool for controlling the plastic behaviour of structures is the application of the static and kinematic theorems of shakedown proposed by Melan [1] and Koiter [2], respectively. These two theorems have been successfully applied to the solution of a large number of problems (see e.g. Maier [3]; Polizzotto [4]; Konig [5], Nina et al. [6] and Simon and Weichert [7]). Evaluate of the lateral load capacity is an important component in the analysis and design of pile foundations subjected to lateral loadings and soil movements. Elastic–plastic solutions for laterally loaded piles were developed recently by Guo [8-9], Qin and Guo [10] and Keawsawasvong and Ukritchon [11]. Depending upon the pile-soil characteristics and the magnitude and type of cyclic loading, the pile response may shakedown and stabilize to an elastic response, or continue to accumulate deflections and deteriorate until failure occurs (Swane and Polus [12]).

In the application of the plastic analysis and design methods the control of the plastic

*Corresponding author: Department of Structural and Geotechnical Engineering, Széchenyi István University Egyetem Tér1, H-9026 Győr, Hungary
majidmr@sze.hu (M. Movahedi Rad)

behaviour of the structures is an important requirement. Since the plastic analysis provides no information about the magnitude of the plastic deformations and residual displacements accumulated before the adaptation of the structure, therefore for their determination several bounding theorems and approximate methods have been proposed. Among others Kaliszky and Lógó [13], Movahedi and Lógó [14] and Movahedi [15] suggested that the complementary strain energy of the residual forces could be considered as an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a limit for the magnitude of this energy.

2. MECHANICAL MODELLING

2.1 Failure mechanisms

Short and long piles fail under different mechanisms. A short rigid pile, unrestrained at the head, tends to rotate or tilts as shown in Fig. 1a and passive resistance develops above and below the point of rotation on opposite sides of the pile. For long pile, the passive resistance is very large and pile cannot rotate or tilt. The lower portion remains almost vertical due to fixity while the upper part deflects in flexure. The pile fails when a plastic hinge is formed at the point of maximum bending moment as shown in Fig. 1b, long pile fails when the moment capacity is exceeded (structural failure).

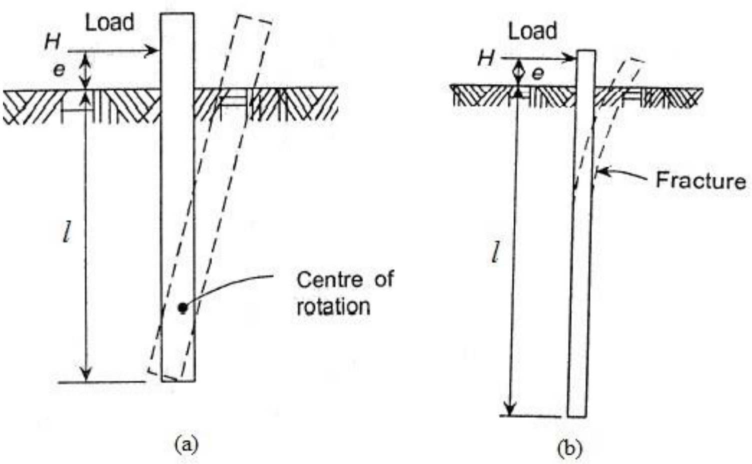


Figure 1. Failure mechanisms of pile under horizontal load: (a) short rigid pile, (b) long pile

Assuming a uniform pile cross section for a long pile, a plastic hinge with a moment of M^p will develop at the point of maximum bending moment that has no shear force, i.e. at point of failure in Fig. 2. Pile under the lateral loading has a virtual lateral velocity V, V_0 at the pile head. The lateral velocity at any depth along the pile is assumed decreasing linearly from V_0 to 0 at point of failure and can be expressed as:

$$V = V_0(1 - \frac{Z}{l}) \tag{1}$$

where Z is the depth measured from pile head, l is the depth where plastic hinge forms. This mechanism was originally proposed by Murff and Hamilton [16]. It is assumed that the lateral soil resistance is fully developed at the ultimate state. The ultimate soil resistance is described by the generic limiting force profile (LFP) proposed by Guo [17].

$$P_u = A_r(Z + \alpha_0)^n \tag{2}$$

where $P_u =$ ultimate soil resistance or limiting force per unit length; $A_r = S_u N_g d^{1-n}$ (cohesive soil) and $\gamma^{'}\ N_g d^{2-n}$ (cohesionless soil), gradient of the limiting force profile; $d =$ the outer diameter of the pile; $\alpha_0 =$ an equivalent depth to consider the resistance at the ground surface, and $n(< 3) =$ the power governing the shape of the limiting force profile shown in Fig. 3, the values of $n = 0.7$ and 1.7 are generally sufficient accurate for piles in clay and sand; $Z =$ depth below the ground level; S_u average undrained shear strength of cohesive soil; $\gamma_s^{'}$ effective unit weight of overburden soil (i.e. dry weight above water table and buoyant weight below); N_g gradient to correlate clay strength or sand weight with the ultimate resistance P_u. The magnitude of the three input parameters α_0, N_g and n are independent of load levels over the entire loading regime.

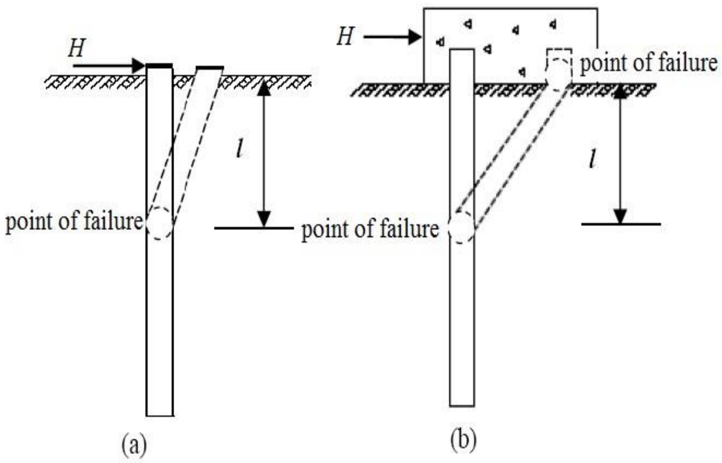


Figure 2. Failure mechanism (a) free-head long pile (b) fixed-head long pile

Guidelines for determining the values of the parameters are discussed by Guo [8] and [18]. The generic limiting force profile (LFP) becomes that suggested for sand by Broms [19], and Barton [20], and that for clay by Matlock [21] and Reese et al. [22], by choosing an appropriate set of $\alpha_0, N_g$and n. For example, selecting$N_g = 3Kp$, $\alpha_0 = 0$ and n = 1, $K_p =$ the coefficient of passive earth pressure, the limiting force profile becomes the Broms' [19] LFP for sand, while giving $\alpha_0 = 2d/N_g$, $N_g = \frac{\gamma_s^{'}\ d}{S_u} + 0.5$, and n = 1, it reduces to Matlock's [21] LFP for soft clay. Here the virtual velocity V_0will be cancelled. The best solution, i.e. the largest load, is found by maximizing the load $H_u$with respect to the optimization parameter l. The details of calculations for plastic limit analysis of lateral piles are explained by Guo [17] and Qin et al. [23]. The solution for free-head long piles are presented below:

$$l = \left[\alpha_0^{n+1} + (n+1)\frac{H_u}{A_r}\right]^{\frac{1}{n+1}} - \alpha_0 \tag{3}$$

the lateral load capacity can be calculated by:

$$\frac{M^p}{A_r} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{H_u}{A_r}\right]^{n+1} - \left[\frac{\alpha_0^{n+2}}{n+2} + \alpha_0\frac{H_u}{A_r}\right] \tag{4}$$

The influence of the loading eccentricity may be considered by replacing the plastic moment $M^p$with M_0, where $M_0 = H_u e$, e is the eccentricity. Consequently:

$$\frac{M^p}{A_r} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{H_u}{A_r}\right]^{\frac{n+2}{n+1}} - \left[\frac{\alpha_0^{n+2}}{n+2} + \alpha_0\frac{H_u}{A_r}\right] + \frac{H_u e}{A_r} \tag{5}$$

For the case of a fixed-head pile, the energy dissipation due to the plastic moment M^p at the failure point is calculated. Following the same calculations as for the free-head piles, the ultimate lateral capacity for fixed-head piles can be easily determined, Guo [17] and Qin et al. [23].

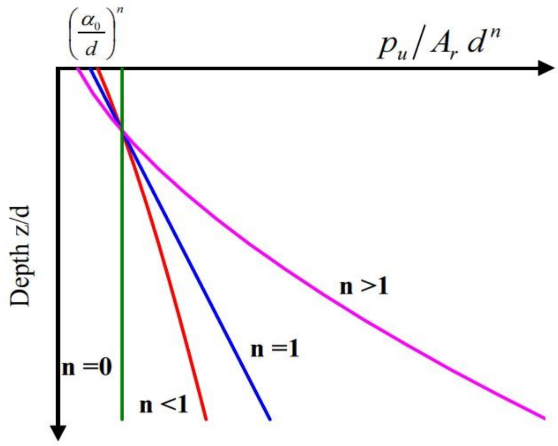


Figure 3. Schematic generic limiting force profiles

2.2 Loadings

The structure is subjected to two independent loads $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ with multipliers $m_1 \geq 0, m_2 \geq 0$ (Fig. 4). In the analysis five loading cases ($h = 1,2, \dots ,5$) shown in Table 1 are taken into consideration. For each loading case a shakedown load multiplier m_{sh} can be calculated. Making use of these multipliers a limit curve can be constructed in the plane m_1/l and m_2/l (Fig. 5). Structure does not fail, under the action of the loads $m_1\boldsymbol{P}_1$and $m_2\boldsymbol{P}_2$, if the points corresponding to the multipliers m_1/l and m_2/l lies inside or on the limit curve.

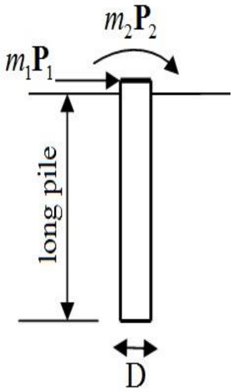


Figure 4. Loads on the pile

Table 1: Load combinations

h	Multipliers	Loads	Load multipliers
1	$m_2 = 0$	$\boldsymbol{Q}_1 = \boldsymbol{P}_1$	m_{sh1}
2	$m_1 = 0$	$\boldsymbol{Q}_2 = \boldsymbol{P}_2$	m_{sh2}
3	$m_1 = 0.5m_2$	$\boldsymbol{Q}_3 = [0.5\boldsymbol{P}_1, \boldsymbol{P}_2]$	m_{sh3}
4	$m_1 = m_2$	$\boldsymbol{Q}_4 = [\boldsymbol{P}_1, \boldsymbol{P}_2]$	m_{sh4}
5	$m_1 = 2m_2$	$\boldsymbol{Q}_4 = [\boldsymbol{2P}_1, \boldsymbol{P}_2\,]$	m_{sh5}

2.3 Control of the plastic deformations

At the application of the plastic analysis and design methods the control of the plastic behaviour of the structures is an important requirement. Following the suggestions of Kaliszky and Lógó [13], Movahedi and Lógó [14] and Movahedi [15] the complementary strain energy of the residual forces could be considered as an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a bound for the magnitude of this energy:

$$\frac{1}{2}\sum_{i=1}^{n} \boldsymbol{Q}_i^r\, F_i \boldsymbol{Q}_i^r \leq W_{p0} \tag{6}$$

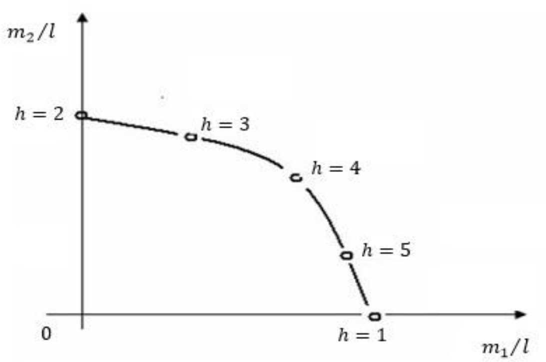


Figure 5. Limit curve and safe domain

Here W_{p0} is an assumed bound for the complementary strain energy of the residual forces and $\boldsymbol{Q}^r$ residual internal forces. This constraint can be expressed in terms of the residual moments $M_{i,a}^r$ and $M_{i,b}^r$ acting at the ends (a and b) of the finite elements as:

$$\frac{1}{6E}\sum_{i=1}^{n}\frac{l_i}{I_i}\left[\left(M_{i,a}^r\right)^2+\left(M_{i,a}^r\right)\left(M_{i,b}^r\right)+\left(M_{i,b}^r\right)^2\right]\leq W_{p0} \tag{7}$$

By the use of (7) a limit state function can be constructed:

$$g\left(W_{p0,}M^r\right)=W_{p0}-\frac{1}{6E}\sum_{i=1}^{n}\frac{l_i}{I_i}\left[\left(M_{i,a}^r\right)^2+\left(M_{i,a}^r\right)\left(M_{i,b}^r\right)+\left(M_{i,b}^r\right)^2\right]. \tag{8}$$

The plastic deformations are controlled while the bound for the magnitude of the complementary strain energy of the residual forces exceeds the calculated value of the complementary strain energy of the residual forces.

3. SHAKEDOWN ANALYSIS

The solution method based on the shakedown theorem which is formulated as below:

$$\text{Maximize } m_{sh} \tag{9a}$$

subject to

$$\boldsymbol{G}^*M^r=0; \tag{9b}$$

$$M^e=\boldsymbol{F}^{-1}\boldsymbol{G}\boldsymbol{K}^{-1}m_{sh}\boldsymbol{Q}_h; \tag{9c}$$

$$M^r+maxM^e\leq M^p \tag{9d}$$

$$\frac{1}{6E}\sum_{i=1}^{n}\frac{l_i}{I_i}\left[\left(M_{i,a}^r\right)^2+\left(M_{i,a}^r\right)\left(M_{i,b}^r\right)+\left(M_{i,b}^r\right)^2\right]\leq W_{p0}. \tag{9e}$$

Here $\boldsymbol{F},\boldsymbol{K},\boldsymbol{G},\boldsymbol{G}^*$: flexibility, stiffness, geometrical and equilibrium matrices, respectively. Eq. (9.b) is an equilibrium equation for the residual moment, M^r. Eq. (9.c) express the calculations of the elastic fictitious moments, M^e. Eq. (9.d) is used as yield conditions. Eq. (9.e) is used to control the plastic deformations. This is a mathematical programming problem which can be solved by the use of nonlinear algorithm. Selecting one of the loading combination $\boldsymbol{Q}_h$; $(h=1,2,\ldots,5)$ a shakedown load multiplier m_{sh} can be determined, then the limit curve of the plastic limit state can be constructed.

4. NUMERICAL EXAMPLES

To demonstrate the theories and solution strategy introduced above, a nonlinear mathematical programming procedure is elaborated where one has to determine the safe loading domain of a laterally loaded long pile with deterministic loading data and with bound for the magnitude of the complementary strain energy of the residual forces.

The application of the method is illustrated by two examples. The first example shows a free-head steel long pile subjected to a lateral load and bending moment at its top with diameter of $D = 20cm$ in cohesionless soil (Fig. 6). The working loads are $\boldsymbol{P}_1 = \mathbf{H} = 10KN$, $\boldsymbol{P}_2 = \mathbf{M} = 4KNm$. The yield stress and the Young's modulus are $\sigma_y = 21KN/cm^2$ and $E = 2.06 \cdot 10^4 KN/cm^2$. The second example shows a fixed-head steel long pile subjected to a lateral load and bending moment at its top with diameter of $D = 20cm$ in cohesionless soil (Fig. 7). The working loads are $\boldsymbol{P}_1 = \mathbf{H} = 10KN$, $\boldsymbol{P}_2 = \mathbf{M} = 5KNm$. The yield stress and the Young's modulus are $\sigma_y = 21KN/cm^2$ and $E = 2.06 \cdot 10^4 KN/cm^2$.

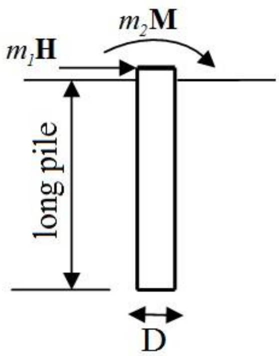


Figure 6. Loads on the free-head pile

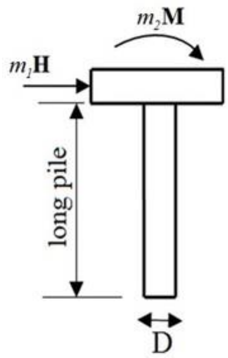


Figure 7. Loads on the fixed-head pile

The results of the solution technique for free and fixed-head steel long piles are presented in Fig. 8 and Fig. 9 respectively, where deterministic loading is considered. The results are in very good agreement with the expectations. In the figures the safe limit load domains are presented in case of different complementary strain energy of the residual forces ($W_{p0} = 40; 45; 50; 55$). One can see that increasing the complementary strain energy of the residual forces results bigger safe loading domain.

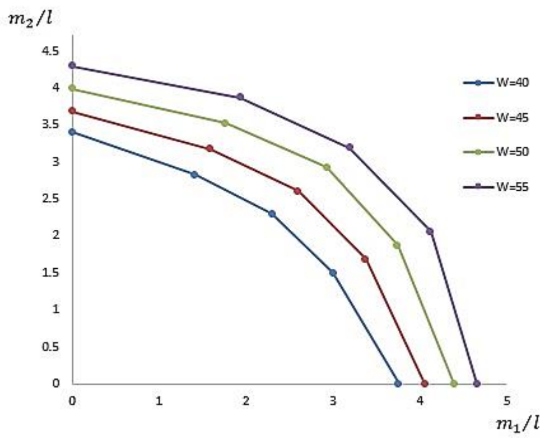


Figure 8. Safe loading domain for free-head steel long pile

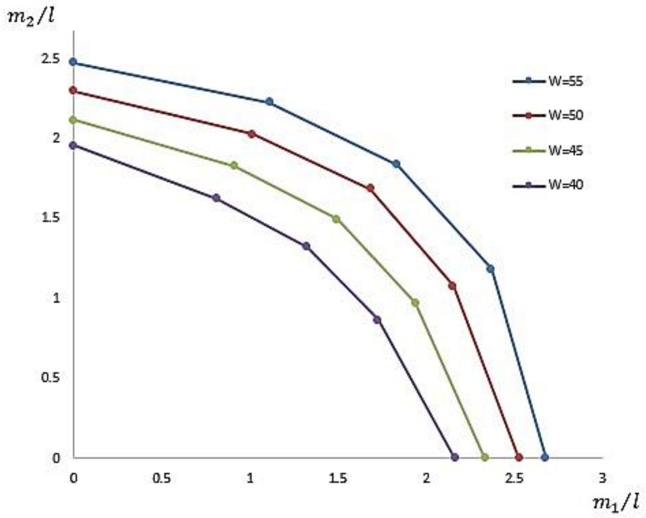


Figure 9. Safe loading domain for fixed-head steel long pile

5. CONCLUSIONS

In the paper shakedown analysis of laterally loaded pile foundation with limited residual strain energy capacity is studied by an appropriate model. Limit curves are presented for the shakedown multipliers. The numerical analysis shows that the bound of the complementary strain energy of the residual forces can influence significantly the magnitude of the shakedown multipliers. The presented investigation drowns the attention to the importance of the problem but further investigations are necessary to make more general statements.

Acknowledgment: The research described in this paper was financially supported by the Hungarian Human Resources Development Operational Programme (EFOP-3.6.1-16-2016-00017).

REFERENCES

1. Melan E. Zur Plastizität des räumlichen Kontinuums, (in German), *Ingenieur-Archiv* 1938; **9**: 116–26.
2. Koiter WT. General theorems for elastic-plastic solids, *Progress Solid Mech* 1960; **1**: 167-221.
3. Maier G. Complementary plastic work theorems in piecewise-linear elastoplasticity, *Int J Solid Struct* 1969; **5:** 261-70.
4. Polizzotto C. A unified treatment of shakedown theory and related bounding techniques, *Solid Mech Arch* 1982; **7**: 19-75.
5. Konig JA. *Shakedown of Elastic-Plastic Structures (Fundamental Studies in Engineering),* Elsevier Publishing Company, Warsaw, Poland, 1987.

6. Nina HL, Itai E, Tim H. Cyclic shakedown of piles subjected to two-dimensional lateral loading, *Int J Numer Analy Meth Geomech* 2009; **33**(10): 1339–61.
7. Simon JW, Weichert D. Shakedown analysis of engineering structures with limited kinematical hardening, *Int J Solid Struct* 2012; **49**: 2177–86.
8. Guo WD. Simple model for nonlinear response of 52 laterally loaded piles, *J Geotech Geoenviron Eng, ASCE* 2013; **139**(2): 234-52.
9. Guo WD. Laterally loaded rigid piles in coheionless soil, *Canadian Geotechnical Journal* 2008; **45**(5): 676-97.
10. Qin HY, Guo WD. Nonlinear response of laterally loaded rigid piles in sand, *Geomech Eng* 2014; **7**(6): 679–703.
11. Keawsawasvong S, Ukritchon B. Ultimate lateral capacity of two dimensional plane strain rectangular pile in clay, *Geomech Eng* 2016; **11**(2) 235-52.
12. Swane IC, Poulos HG. Shakedown analysis of laterally loaded pile tested in stiff clay, *In: 4th Australia–New Zealand Conference on Geomechanics*, Perth, Australia, 1984, pp. 165-169.
13. Kaliszky S, Lógó J. Optimal plastic limit and shakedown design of bar structures with constraints on plastic deformation, *Eng Struct* 1997; **19**(1): 19-27.
14. Movahedi RM, Lógó J. Plastic behavior and stability constraints in the reliability based shakedown analysis and optimal design of skeletal structures, *Asian J Civil Eng* 2011; **12**(4): 395-413.
15. Movahedi RM. Reliability based analysis and optimum design of laterally loaded piles, *Periodica Polytech Civil Eng* 2017; **61**(3): 491-97.
16. Murff JD, Hamilton JM. P-ultimate for undrained analysis of laterally loaded piles, *J Geotech Eng, ASCE* 1993; **119**(1): 91-107.
17. Guo WD. On limiting force profile, slip depth and response of lateral piles, *Comput Geotech* 2006; **33**(1): 47-67.
18. Guo WD. *Theory and Practice of Pile Foundations*, Taylor and Francis, London, UK, 2012.
19. Broms BB. Lateral resistance of piles in cohesiveless soils, *J Soil Mech Found Eng, ASCE* 1964; **90**(3): 123-56.
20. Barton YO. *Laterally Loaded Model Piles in Sand: Centrifuge Tests and Finite Element Analysis*, Ph.D. Dissertation, University of Cambridge, Cambridge, UK, 1982.
21. Matlock H. Correlations for design of laterally loaded piles in soft clay, *Proceedings of Second Annual Offshore Technology Conference*, Houston, USA, 1970, pp. 577-94.
22. Reese LC, Cox WR, and Koop FD. Field testing and analysis of laterally loaded piles in stiff clay, *Proceeding of the 7th Annual OTC*, Houston, USA, 1975, pp. 672-90.
23. Qin HY, Oh EY, Guo WD and Dai PF. Upper bound limit analysis of lateral pile capacity, *Proceedings of Pile 2013: International Conference on State of the Art of Pile Foundation and Pile Case Histories*, Bandung, Indonesia, 2013, pp. 1-8.