



NEW OPTIMIZED EQUATIONS WITH INTELLIGENT MODELS FOR PREDICTING HYDRAULIC JUMP CHARACTERISTICS OVER ARTIFICIAL AND NATURAL ROUGH BEDS

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ABSTRACT

The available studies for estimating the characteristics of hydraulic jump are only for artificial or natural beds, and very limited researches have simultaneously considered artificial and natural beds. The aim of this study is to present comprehensive equations and models for predicting the characteristics of hydraulic jump in artificial and natural rough beds with various dimensions, arrangement and roughness forms. The experimental data of different researches on two artificial and natural rough beds (containing 559 data series) were collected. After randomization, the data were used in combination of 75-25 for training and testing the two intelligent models of K-nearest neighbors (KNN) and M5 model tree with various scenarios and their performance were evaluated in estimation of hydraulic jump characteristics (including sequent depth, energy loss and shear force coefficient). Then, the existing empirical equations examined and calibrated and new optimized equations were derived using Solver command in Excel software. The results of the best intelligent models were analyzed and compared with the best calibrated and new optimized equations. Both the intelligent models had the same performance. In the M5 model tree, the best scenario of all the three parameters of sequent depth ($R^2=0.90$), energy loss ($R^2=0.94$), and shear force coefficient ($R^2=0.81$) obtained by using Froude number as input parameter. The best empirical equations were Abbaspour et al.'s ($R^2=0.90$), Abbaspour and Farsadzadeh's ($R^2=0.90$), and Akib et al.'s ($R^2=0.83$) for the sequent depth, the energy loss and the shear force coefficient, respectively. The calibrated and new optimized equations had a similar precision as the intelligent models, but their errors were less than that of the best empirical equations.

Keywords: energy loss; M5 model tree; optimized equations; sequent depth; shear force coefficient.

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1. INTRODUCTION

Hydraulic jump is one of the most important hydraulic principles of open channel flow which is caused by a change in flow regime from supercritical to subcritical condition. This phenomenon is a rapidly varied flow and generally occurs at downstream of the gates and spillways. In these structures, a stilling basin is necessary to dissipate the excessive energy of the flow through them. In order to avoid the damage to downstream structures, it is recommended to limit the hydraulic jump through the stilling basin. In the other hand, to reduce the construction cost of the basin, the use of various bed roughnesses should be considered [1]. The roughness bed can be in the form of a sinusoidal wave, trapezoidal, triangular, and rectangular (as the artificial bed) and gravel particles (as the natural bed). In recent years, this phenomenon has been extensively investigated by researchers in different condition to achieve the influences of roughness on the hydraulic jump characteristics. Fig.1 shows the characteristics of hydraulic jump on artificial and natural rough beds. In this figure, y_1 and y_2 are respectively the initial and sequent hydraulic jump, L_j and L_r are respectively the jump length and the rolling length, and k_s is the height roughness.

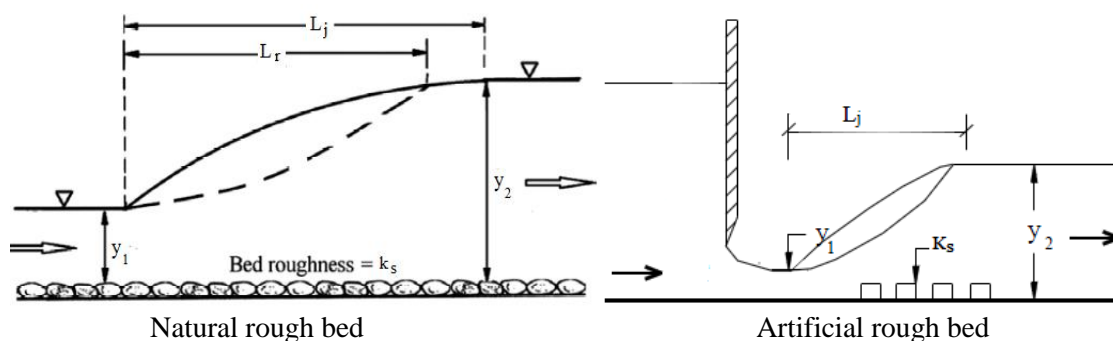


Figure 1. Characteristics of hydraulic jump on the rough bed

Numerous investigations have been conducted on the effect of roughness on hydraulic jump characteristics. Ead and Rajaratnam [2] by using sinusoidal rough beds indicated that the jump length on corrugated beds is one half of its length over smooth beds. Carolo et al. [3] performed some experiments on hydraulic jump over natural rough bed with different diameters of gravels and cobbles. They suggested equations for estimating the relative sequent depth and rolling length. Deshpande et al. [4] investigated the effect of spaced and staggered semi-circular strip corrugated beds on characteristics of hydraulic jump and concluded that, for two beds, the sequent depths were reduced by 29% and 34%, respectively, and the jump length were reduced by 21% and 24%, respectively. Ghorbani et al. [5] investigated the characteristics of hydraulic jump over a natural rough bed with two different diameters. The results indicated that by increasing the diameter from 4.45 to 5.75 mm, the jump length decreases by 13.5%, but the gravel size has no significant effect on the sequent depth. Asadi et al. [6] studied the characteristics of hydraulic jump over natural rough bed and obtained the sequent depth, energy loss and shear force coefficient as functions of Froude number. Parsamehr et al. [7] investigated the characteristics of hydraulic jump over rough bed with discontinuous elements of lozenge. The results showed that maximum reduction of the sequent depth was 29.39% and the increase of the energy

dissipation and the bed shear stress coefficient on rough bed were 10.94% and 13.54%, respectively, relative to corresponding value on smooth bed.

In the recent years, application of the intelligent models in predicting the behavior of various water engineering phenomena has been increased. The advantages of these models are the mathematical expression of physical phenomena and providing a good estimation of the unknown parameters in the model by using a small data collection. Nowadays, intelligent models namely artificial neural network (ANN), K-nearest neighbors (KNN) and decision tree (DT) are considered in simulating water engineering phenomena. M5 model, a model tree, is used for predicting the continuous numerical attributes in which linear regression functions appear in the leaves of this tree. The results of M5 model tree are easy to understand and simulate and its output has a high accuracy that can be compared with other models [8]. The K-nearest neighbors (KNN) is a nonparametric method that operates using the principle of similarity and proximity of data. In the recent years, several studies have been carried out by using different soft computing methods, including M5 model tree and K-nearest neighbors (KNN) for investigating hydraulic phenomena [9]. Some of the researches conducted about the hydraulic jump by using intelligent models are Negam [10] with artificial neural network and multiple linear regression models, Abbaspour et al. [11] with ANN and genetic programming (GP) models, Karbasi and Azamatulla [12] with gene expression programming (GEP), ANN and support vector regression (SVR), and Roknian and Heydari [13] with artificial neural network (ANN).

In the previous studies, the characteristics of hydraulic jump over artificial and natural rough beds with different dimensions, arrangements and shapes have been extensively investigated by researchers. In these studies, a number of equations have been provided for estimation of hydraulic jump characteristics with some limitations or considerable errors. In this study, the performance of intelligent models including the M5 model tree and K-nearest neighbors (KNN) for estimating the sequent depth and energy loss and shear force coefficient was investigated. For this purpose, available laboratory data of different researches on artificial and natural rough beds (named total bed) were used. Also, by evaluating the researchers' empirical equations, some new optimized equations were provided using optimization method. Finally, the best soft computing model as well as optimized-regression equations was introduced for estimating the hydraulic jump characteristics.

2. MATERIALS AND METHODS

In this study, the new available experimental data of various researches on the hydraulic jumps over artificial and natural rough beds (considered as total bed) were used. Table 1 shows the experimental hydraulic jump data and the range of parameters. The data used in this study were 559 data series including upstream Froude number (Fr), relative sequent depth (y_2/y_1) and relative roughness (k_s/y_1), shear force coefficient (ϵ) and relative energy loss ($\Delta E/E_1$). It is notified that in Carollo et al.'s study [3], the rolling length (L_r) has been used instead of jump length (L_j), thus, these two parameters were not investigated in the total bed.

Table 1: The experimental data of hydraulic jump on artificial and natural rough beds

Type of rough bed	Researchers	k_s/y_1	F_r	y_2/y_1	$\Delta E/E_1$	ε
Artificial rough	Ead and Rajaratnam [2]	0.26-0.51	4-10	4.09-10.35	0.47-0.78	7.96-73.19
	Evcimen [14]	0.29-0.93	7.29-15.91	8.7-18.04	0.67-0.85	19.18-170.55
	Simsek [15]	0.26-0.71	2.13-11.92	2.59-14.8	0.10-0.79	0.45-56.8
	Abbaspour et al. [16]	0.25-1.16	4-8	4.13-8.5	0.47-0.72	5.23-39.17
	Elsebaie and Shabayek [17]	0.36,0.72	3-7.5	2.2-6.4	0.40-0.77	5.37-59.47
	Evcimen [18]	0.37-2.2	3.92-13.28	4.16-14.92	0.42-0.85	1.17-158.39
Natural rough	Carollo et al. [3]	0.1-2	1.9-9.9	2.82-9.72	0.08-0.82	0.08-106.50

At first, the data were randomized by using the Kutools of Excel software. Then, by using the WEKA software, performance of M5 model tree and the K-nearest neighbor algorithm in different scenarios to estimate the hydraulic jumps parameters was evaluated. For this purpose, the data combination of 75-25 (75% of data for training and the reminded for testing) was used. Also, the performance of existing empirical equations in estimating relative sequent depth, energy loss and shear force coefficient was investigated and the best equations were introduced. By the same data combination, the previous equations were calibrated. Then, by using the Solver command in the Excel software, various forms of regression equation were evaluated and some new optimized equations were derived. The Solver can be used to minimize the sum of squares of residuals (differences between observed and calculated values), and to perform least-squares curve fitting. More details about the optimization method of the Solver command is given in Billo [20]. The best model or new optimized equations to estimate the hydraulic jump characteristics were selected based on the highest R^2 and the lowest RMSE (also, based on the minimum number of rules in M5 model tree). Finally, by comparing the performance of the best soft computing model scenarios with the new optimized equations, the best model or equation for estimation of hydraulic jump characteristics was chosen.

M5 model tree introduced by Quinlan [21] is a subset of machine learning and data mining models. Tree-based models are one of data mining techniques in which output is a decision tree. The structure of decision tree is like a tree composed of roots, branches, nodes and leaves. The decision tree is depicted from top to bottom. The root, as the first node, is placed on top and the chain of branches and nodes ends on the leaves. Each node is related to a predictor variable, and branches are performed in the node, and the branching intervals are chosen so that the sum of the squares of root mean deviations reaches to minimum [16]. The process of branching in each node is repeated until reaches to the end node (leaf). Finally, a large tree is developed which is pruned and straightened to achieve an optimal and efficient tree.

The KNN is a nonparametric method used for classification and regression. KNN algorithm computes a weighted average of the KNNs, which are weighted by the inverse of their distance. The algorithm computes a distance between the query example and the labeled examples and orders the labeled examples in increasing distance. A case is classified by a majority vote of its neighbors, with the case being assigned to the class most common among its KNN measured by a function. To calculate the distance between each new sample from observational samples, distance functions such Hamming, Euclidian, and Chebisheph are used. In this study, the Euclidian function was used. In the KNN algorithm, in order to

achieve the best result, determining the optimal value of the K parameter is of great importance. For this purpose, the trial and error method is used [22]. In the present study, WEKA software, developed at Waikato University in New Zealand, was used for performance of M5 model tree and the K-nearest neighbors algorithm (KNN).

3. RESULTS AND DISCUSSION

3.1 Intelligent models

The results of M5 model tree and K-nearest neighbors (KNN) for estimating the relative sequent depth (y_2/y_1), relative energy loss ($\Delta E/E_1$) and shear force coefficient (ϵ) in different scenarios are presented in Table 2. According to the results, both intelligent models have almost the same performance, so that there is no significant difference in the accuracy and error of their corresponding scenarios. Negam [10] compared the performance of artificial neural network with linear regression and reported that accuracy of artificial neural network is better than linear regression. Abbaspour et al. [11] simulated the hydraulic jump on rough bed by using artificial neural network (ANN) and genetic programming (GP). The results showed that there is a good agreement between observed and predicted values by neural network and genetic programming. Karbasi and Azamatulla [12] by evaluating the performance of intelligent methods of the gene expression programming (GEP), artificial neural network (ANN) and support vector regression (SVR), stated that all the three models have an approximately similar accuracy in estimating the characteristics of hydraulic jump in rough beds.

According to the results of different scenarios for each of three parameters, the scenario with Froude number (Fr) as input was selected as the practical scenario with a high R^2 , an appropriate RMSE, and with minimum input parameter and number of rules. It can be said that the practical scenarios for estimating sequent depth (scenario 2) and energy loss (scenario 4) have a more acceptable performance than the scenario with all input parameters (Scenario 1). In shear force coefficient parameter, the scenario with the input parameters of the Froude number and relative sequent depth (scenario 1) has the best performance. But, in this scenario the number of rules is high (9 rules) and in the empirical equations for estimating shear force coefficient, only the Froude number has been used. So the scenario 4 has been chosen as the practical scenario. Almost all researchers, such as Evcimen (14, 19), Deshpand et al. [4], Abbaspour and FarsadiZadeh [23], Ead and Rajaratnam [2], Izadjo and Shafae Bajestan [24], Tokyay et al. [25], Asadi et al. [6] and Akib et al. [26] have used only the Froude number in empirical equations to estimate the sequent depth, energy loss and shear force coefficient.

In each the three parameters, relative roughness (k_s/y_1) has the lowest R^2 , which indicates the low effect of this parameter in estimation of hydraulic jump characteristics. In numerous studies such as Ead and Rajaratnam [2], Abbaspour et al. [17], Tokyay et al. [25], Parsamehr and Hosseinzadeh Dalir [7] and Ghorbani et al. [5], it has been concluded that relative roughness has no impressive effect in estimation of hydraulic jump characteristics. In the following, based on the decision tree and linear models produced by M5 model tree, the results of the practical scenarios of this model are presented for all the three parameters.

For the sequent depth, the practical model of M5 (scenario 2) has no decision tree and

presents only a linear equation. In other words, the equation 1 is sufficiently used for all range of the Froude number on rough beds (artificial and natural). Many researchers have provided a linear equation with Froude number to estimate the sequent depth, which have good agreement with the form of the equation of M5 model tree.

$$\frac{y_2}{y_1} = (1.0928 \times Fr) + 0.1324 \quad (1)$$

Table 2: The results of M5 model tree and K-nearest neighbors (KNN) in different scenarios for estimating y_2/y_1 , $\Delta E/E_1$ and ε

Parameter	Number of scenarios	Input parameters	KNN		M5		
			R ²	RMSE	R ²	RMSE	Number of rules
Relative sequent depth (y_2/y_1)	1	$k_s/y_1, Fr$	0.93	0.70	0.92	0.77	1
	2	Fr (best scenario)	0.89	0.86	0.90	0.83	1
	3	k_s/y_1	0.23	2.46	0.25	2.30	3
	1	$k_s/y_1, y_2/y_1, Fr$	0.97	0.03	0.99	0.02	1
	2	$y_2/y_1, Fr$	0.99	0.01	0.99	0.02	1
Relative energy loss ($\Delta E/E_1$)	3	$k_s/y_1, Fr$	0.95	0.04	0.95	0.03	1
	4	Fr (best scenario)	0.94	0.004	0.94	0.04	1
	5	$k_s/y_1, y_2/y_1$	0.83	0.06	0.82	0.07	1
	6	y_2/y_1	0.54	0.11	0.60	0.10	1
	7	k_s/y_1	0.36	0.13	0.41	0.12	5
Shear force coefficient (ε)	1	$y_2/y_1, Fr$	0.93	7.39	0.89	10.40	9
	2	$k_s/y_1, y_2/y_1, Fr$	0.87	10.43	0.88	10.67	8
	3	$k_s/y_1, Fr$	0.83	11.93	0.83	10.00	2
	4	Fr (best scenario)	0.72	15.90	0.81	12.62	2
	5	$k_s/y_1, y_2/y_1$	0.70	15.97	0.73	15.07	11
	6	y_2/y_1	0.52	22.01	0.62	17.91	3
	7	k_s/y_1	0.40	22.49	0.27	24.79	6

For the energy loss, Fig. 2 presents the structure of decision tree for the practical scenario (scenario 4). According to this figure, the practical scenario provides 5 linear equations (Table 3). The first linear equation belongs to the Froude numbers less than 3.9. The next equations are related to the $3.9 < Fr < 4.9$, $4.9 < Fr < 5.5$, $5.5 < Fr < 7.5$ and $Fr > 7.5$. Comparison of the highest Froude number ($Fr=7.5$) with $Fr=9$ belonged to the strong hydraulic jumps over smooth bed shows that classification of hydraulic jumps on rough beds is different from smooth bed. However, it seems that more laboratory studies are necessary to better understand of this issue. According to the linear equations of the relative energy loss (Table 3), the coefficient of Froude number (Fr) has decreased from the LM num 1 to the LM num 5. This indicates that by increasing the Froude number, the energy loss increases asymptotically. Most researchers have reported that when Froude numbers increase, the energy loss asymptotically increases [19, 26].

In Fig. 3, the decision tree of the practical scenario (scenario 4) of M5 for estimating the shear force coefficient is presented. The tree model provides two linear equations for the practical scenario (Table 4). As shown in Fig. 3, the $Fr=6.6$ is turning point. If $Fr < 6.6$, then the LM num 1 is used to calculate the shear force coefficient. Also, if $Fr > 6.6$, then the LM num 2 is used. According to the linear equations (Table 4), it is seen that the coefficient of Fr in equation 2 has doubled with respect to the equation 1, which indicates a significant increase in the shear force for Froude numbers higher than 6.6.

Table 3: Linear equations of the best scenario for $\Delta E/E_1$

Linear model No.	Equations
LM num: 1	$\Delta E/E_1 = (0.1416 \times Fr) - 0.1326$
LM num: 2	$\Delta E/E_1 = (0.053 \times Fr) + 0.2645$
LM num: 3	$\Delta E/E_1 = (0.03 \times Fr) + 0.412$
LM num: 4	$\Delta E/E_1 = (0.0289 \times Fr) + 0.4366$
LM num: 5	$\Delta E/E_1 = (0.0223 \times Fr) + 0.5465$

$Fr \leq 5.5 :$

| $Fr \leq 3.9 : LM1(59 / 34.3\%)$

| $Fr > 3.9 :$

| | $Fr \leq 4.9 : LM2(71 / 24.9\%)$

| | $Fr > 4.9 : LM3(53 / 25.0\%)$

$Fr > 5.5 :$

| $Fr \leq 7.5 : LM4(141 / 20.0\%)$

| $Fr > 7.5 : LM5(123 / 16.2\%)$

Figure 2. Tree structure of the best scenario for $\Delta E/E$ Table 4: Linear equations of the best scenario for ε

Linear model No.	Equations
LM num 1	$\varepsilon = (6.1690 \times Fr) - 19.5965$
LM num 2	$\varepsilon = (11.9904 \times Fr) - 56.277$

$Fr \leq 6.6 : LM1(255 / 24.8\%)$

$Fr > 6.6 : LM2(161 / 66.1\%)$

Figure 3. Tree structure of the best scenario for ε

3.2 Empirical and new optimized equations

In Table 5, the results of available empirical equations as well as calibrated and new optimized equations are presented. According to the multiplicity of empirical equations and different forms of regression equations, only the results of equations with high R^2 and minimum RMSE have been presented. In empirical equations for estimation of sequent depth, because of the same form of equations (linear form), the value of R^2 is equal, and difference in performance is related to their errors (RMSE). Accordingly, Abbaspour et al.'s equation [17] with $R^2=0.90$ and $RMSE=0.83$ was chosen as the best empirical equation. In optimized equations, equation 2 with $R^2=0.90$ and $RMSE=0.81$ was selected as the best regression equation to estimate the sequent depth. It is noteworthy that in the Blanger's modified equation, the coefficient of Froude number has been obtained 5.55, while in a smooth bed it is equal to 8.

In empirical equations for estimating energy loss, Abbaspour and Farsadizadeh's equation [23] with $R^2=0.90$ and $RMSE=0.06$ was selected as the best equation. In the regression equations, it can be seen that the two equations of 4 and 5 have high accuracy and less error than Abbaspour and Farsadizadeh's equation [23]. In general, equation 5 is chosen as the best regression equation for estimating the energy loss.

In the shear force coefficient, Akib et al.'s equation [26] with $R^2=0.83$ and $RMSE=17.49$ was selected as the best empirical equation. According to Table 5, the accuracy of the modified and new optimized equations is similar to empirical equations, but their error has decreased significantly. According the results, Akib et al.'s modified equation with $R^2=0.83$ and $RMSE=12.24$ was chosen as the best regression equation for calculation of the shear force coefficient.

In Fig. 4, the fitting curves of the experimental parameters versus the calculated values of the practical scenario of M5 as well as the best empirical, modified and new optimized equations are presented. According to Fig.4a (the fitting curve of sequent depth), there is no considerable difference between the performance of M5 and Abbaspour et al.'s equation [17] and the new optimized equation (No. 2). In fact, accuracy and error of these three models are similar to each other. In the energy loss (Fig.4b), the performance of M5 and the new optimized equation (No. 5) is equal to each other and better than that of the best empirical equation (Abbaspour and Farsadizadeh, [23]). Also, according to Fig.4c (shear force coefficient), the performance of M5 and Akib et al.'s modified equation is similar. The proper dispersion of measured and predicted values around $y=x$ line in Fig.4a-c indicates a good accuracy and low error of M5, modified and new optimized equations in estimation of hydraulic jump characteristics, especially in the sequent depth and energy loss. In general, it can be notified that intelligent models, especially M5 model tree by providing simple and accurate linear models and a decision tree, can be used to estimate hydraulic jump characteristics on artificial and natural beds. These linear equations and decision tree are consistent with the physics governing on the phenomenon.

Table 5: The results of empirical equations and new optimized equations for calculation relative sequent depth, relative energy loss and shear force coefficient. (With M5 model tree)

Parameter	Researchers	Equations	R ²	RMSE
Relative sequent depth	Abbaspour et al. [17]	$\frac{y_2}{y_1} = (1.1146 \times Fr)$	0.90	0.83
	Tokyay et al. [25]	$\frac{y_2}{y_1} = (1.1223 \times Fr) + 0.0365$	0.90	0.84
	Izadjo and Shafae Bajestan [24]	$\frac{y_2}{y_1} = (1.047 \times Fr) + 0.5902$	0.90	0.87
	Blanger's modified equation	$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + (5.55 \times Fr^2)} - 1 \right]$	0.90	0.84
	New Eq. 1	$\frac{y_2}{y_1} = \sqrt{(Fr^2 + 2)}$	0.90	0.82
	New Eq. 2	$\frac{y_2}{y_1} = 1 + (0.75 \times Fr)^{1/15}$	0.90	0.81
	Best M5 model tree Scenario	$\frac{y_2}{y_1} = (1.0928 \times Fr) + 0.1324$	0.90	0.83
Relative energy loss	Abbaspour and Farsadizadeh [23]	$\frac{\Delta E}{E_1} = (-0.01 \times Fr^2) + (0.19 \times Fr) - 0.17$	0.90	0.06
	Evcimen [14]	$\frac{\Delta E}{E_1} = (-0.0026 \times Fr^2) + (0.08 \times Fr) + 0.2362$	0.85	0.08
	Deshpand et al. [4]	$\frac{\Delta E}{E_1} = 0.09Fr + 0.15$	0.74	0.14
	New Eq. 3	$\frac{\Delta E}{E_1} = \left(1 - \frac{\sqrt{2}}{Fr} \right)^2$	0.93	0.06
	New Eq. 4	$\frac{\Delta E}{E_1} = \left(1 - \frac{1.28}{Fr} \right)^2$	0.93	0.04
	New Eq. 5	$\frac{\Delta E}{E_1} = \left(1 - \frac{2}{Fr} \right)^{1.14}$	0.94	0.04
	Best M5 model tree Scenario	According to Table 3	0.94	0.04
Shear force coefficient	Akib et al. [26]	$\varepsilon = (0.405 \times Fr^2) - (0.253 \times Fr)$	0.83	17.49
	Ead and Rajaratnam [2]	$\varepsilon = (Fr - 1)^2$	0.83	19.21
	Abbaspour and Farsadizadeh [23]	$\varepsilon = (1.33 \times Fr^2) - (4.66 \times Fr) + 7.7$	0.83	32.03
	Ead and Rajaratnam's modified equation	$\varepsilon = (Fr - 1)^{1.866}$	0.83	12.25
	New Eq. 6	$\varepsilon = 0.587 \times Fr^2$	0.83	12.44
	Akib et al.'s modified equation	$\varepsilon = (0.749 \times Fr^2) - (1.433 \times Fr)$	0.83	12.24
	Best M5 model tree Scenario	According to Table 4	0.81	12.62

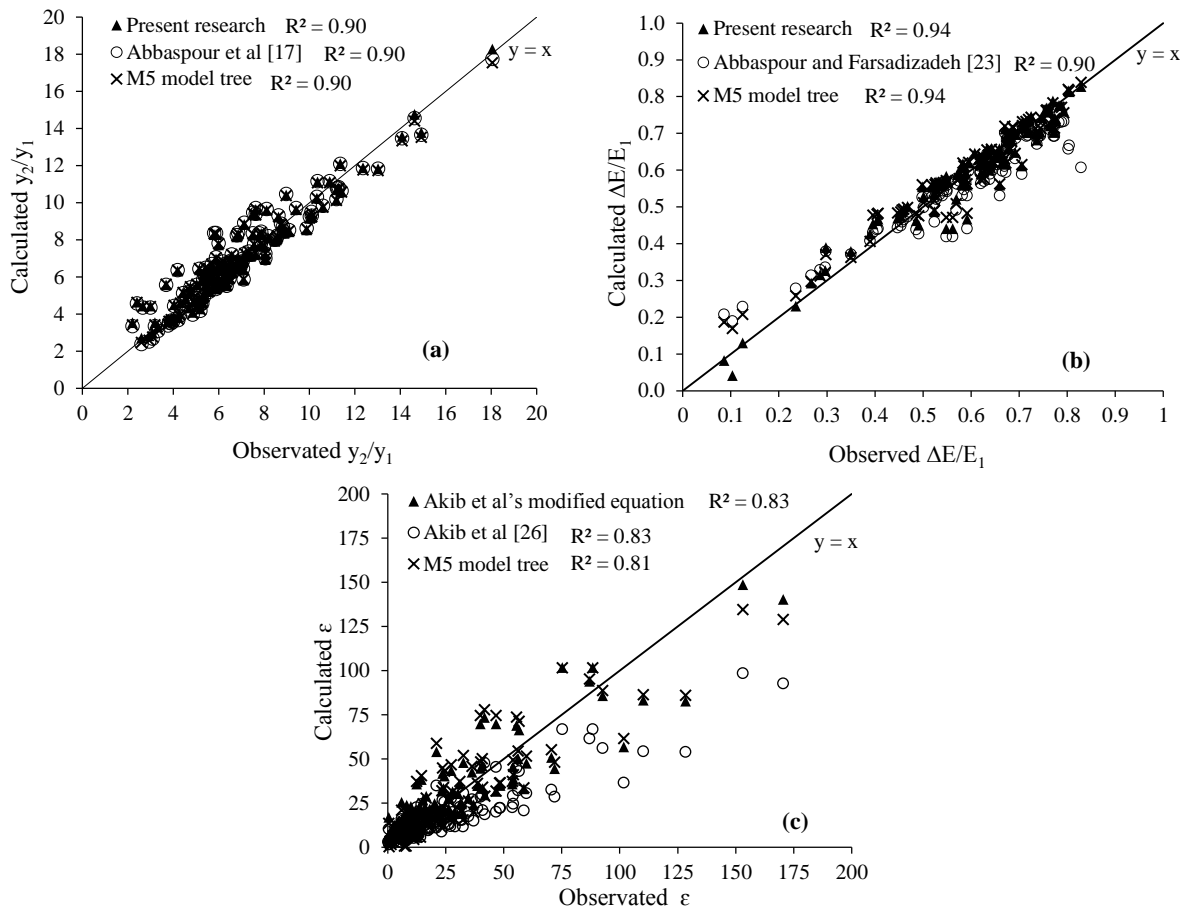


Figure 4. Measured versus predicted values of relative sequent depth, relative energy loss and shear force coefficient

4. CONCLUSION

In this study, the performance of existing empirical and new optimized equations as well as two intelligent models of M5 model tree and K-nearest neighbors (KNN) was evaluated for estimating the hydraulic jump characteristics on artificial and natural beds. The results showed that the two intelligent models have almost the same performance. Also, the results of the practical scenarios of M5 for estimating sequent depth, energy loss and shear force coefficient showed that Froude number is a key parameter in calculation of hydraulic jump characteristics and relative roughness parameter has no considerable effect. In the sequent depth, energy loss and shear force coefficient, the best empirical equations belonged to Abbaspour et al. [17], Abbaspour and Farsadzadeh [23] and Akib et al. [26], respectively. The performance of the best modified and new optimized equations was better than the empirical equations of the energy loss and shear force coefficient. However, in sequent depth, there was no significant difference between the results. Also, the modified and new optimized equations had the same performance as M5 model tree.

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