

DAMAGE IDENTIFICATION BY USING MODAL EXPANSION AND TOPOLOGY OPTIMIZATION IN THREE DIMENSIONAL ELASTICITY PROBLEMS

F. Abdollahi and S.M. Tavakkoli^{*,†}

Civil Engineering Department, Shahrood University of Technology, Shahrood, Iran

ABSTRACT

In this paper, topology optimization is utilized for damage detection in three dimensional elasticity problems. In addition, two mode expansion techniques are used to derive unknown modal data from measured data identified by installed sensors. Damages in the model are assumed as reduction of mass and stiffness in the discretized finite elements. The Solid Isotropic Material with Penalization (SIMP) method is used for parameterizing topology of the structure. Difference between mode shapes of the model and real structure is minimized via a mathematical based algorithm. Analytical sensitivity analysis is performed to obtain derivatives of objective function with respect to the design variables. In order to illustrate the accuracy of the proposed method, four numerical examples are presented.

Keywords: structural health monitoring; mode expansion; damage identification; SIMP; topology optimization.

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1. INTRODUCTION

Structural Health Monitoring (SHM) is known as a new branch of structural engineering science that can identify damage in early stages before it propagates and prevents structural failures. Structural health monitoring mainly consists of two main parts: system identification and damage detection. Dynamic characteristics of the structure are obtained in system identification which are used for damage localization. Due to financial and applicability difficulties, it is not possible to extract data from all degrees of freedoms of the structure. Therefore, by observing some of them and using mode expansion approaches the unmeasured data can be obtained [1, 2]. Damage is usually defined as a stiffness reduction

*Corresponding author: Civil Engineering Department, Shahrood University of Technology, Shahrood, Iran

†E-mail address: s.m.tavakkoli@gmail.com (S.M. Tavakkoli)

in the stiffness matrix of the model finite elements and its intensity is specified by the amount of reduction [3]. This reduction causes changes in dynamic characteristics such as natural frequency and mode shapes of the structure. Comparing these parameters between real structure and the numerical model leads to an optimization problem to minimize the differences and update the model to find the damage location and its severity [4, 5].

In the last few years, optimization has been used as a powerful tool for damage detection process with intention of damage localization and quantification. Two types of optimization methods are commonly used in this field: metaheuristic algorithms and mathematical based methods [1]. In 2002, Hao *et al.* worked on damage detection using genetic algorithm with three different objective functions [6]. Stubbs *et al.* proposed a method based on sensitivity of frequency to damage and achieved reasonable results. They examined a cantilever beam and detected the damage, successfully [7]. In 2012, Gerist *et al.* used sparsest solution to obtain linear system of equations by basis pursuit method for using as first population of genetic algorithm to obtain damage solution [8]. Kaveh *et al.* used charged system search algorithm to improve the performance of damage detection for natural frequencies [9–12]. Naserlavi *et al.* proposed a two-stage method for damage detection of large-scale structures [13]. In 2013, Torkzadeh *et al.* worked on damage detection for large-scaled structures by kinetic and modal strain energies using heuristic particle swarm optimization [14]. Hosseinzadeh *et al.* developed a two-stage method for damage localization and quantification in high-rise shear frames based on the first mode shape slope [15]. In 2014, Fathnejat *et al.* worked on cascade feed forward neural network for damage detection. They used modal strain energy based index in order to reduce computational costs [16]. Kaveh *et al.* utilized several metaheuristic methods such as particle swarm strategy, ray optimization, harmony search algorithm, enhanced colliding bodies optimization, Tug-of-War optimization, cyclical parthenogenesis algorithm and enhanced vibrating particles system algorithm for damage identification in different types of structures [17–23]. Saberi *et al.* worked on damage detection of space structures using residual force method in 2015 [24]. In 2016, Hosseinzadeh *et al.* worked on a new damage index for structural damage identification by means of wavelet residual force [25]. Kaveh *et al.* also carried out research on damage detection of skeletal structures based on charged system search optimization using incomplete modal data [26]. In 2017, Zhang *et al.* used level set optimization for continuum plane stress problems with considering eigenvalues and localized damages with relatively good approximation [27]. In 2018, Fallahian *et al.* worked on damage identification in structures using time domain responded based on differential evolution algorithm [28]. Kaveh *et al.* used a thermal exchange optimization algorithm to identify damage in structures [29]. In 2019, Eslami *et al.* used topology optimization in plane structures by using the SIMP method [30].

This paper aims to use topology optimization for damage detection in three dimensional elasticity problems. To achieve this, it is assumed that mode shapes are known in some specified Degrees Of Freedom (DOF) and the rest are approximated by using mode expansion methods [2]. Afterwards, the optimization problem is solved by gradient based optimization methods to find location and intensity of the damage.

There are different methods for parameterization of topology in structures in the literature, such as homogenization method [3, 31, 32] and solid isotropic material with penalization [33, 34] simply known as SIMP which is used in this article. In SIMP method,

density of elements are considered as the design parameters [35, 36]. If density changes in an element, the stiffness and mass matrices will change. Therefore, the SIMP model can be an appropriate method for damage modeling which is used in this paper.

The outline of the paper is as follows. Mode expansion strategies are discussed in section 2. In section 3, optimization problem is defined and the parameters are introduced. Section 4 is devoted to sensitivity analysis and mathematical formulations. Performance of the method is illustrated via four numerical examples in section 5.

2. MODE EXPANSION APPROACH

Mode expansion techniques are used to estimate all data of structural response from the measured data detected by certain amount of sensors. In this section, two methods for modal expansion are considered: System Equivalent Reduction Expansion Process (SEREP) and dynamic expansion method [2]. In SEREP method a transformation matrix (\mathbf{T}_{SEREP}) is defined according to the measured DOFs. \mathbf{T}_{SEREP} is a linear transform which approximates unmeasured DOFs as below

$$\boldsymbol{\varphi}_m = \boldsymbol{\varphi}_m^a \mathbf{T}_{SEREP} \quad (1)$$

$$\mathbf{T}_{SEREP} = \hat{\boldsymbol{\varphi}}_m^a \boldsymbol{\varphi}_m, \quad \hat{\boldsymbol{\varphi}}_m^a = \left(\boldsymbol{\varphi}_m^{aT} \boldsymbol{\varphi}_m^a \right)^{-1} \boldsymbol{\varphi}_m^{aT} \quad (2)$$

In above equations, $\boldsymbol{\varphi}_m$ is vector of measured mode shapes at specified DOFs. $\boldsymbol{\varphi}_m^a$ contains corresponding mode shapes of the model at measured DOFs. \mathbf{T}_{SEREP} is transformation matrix used to convert unmeasured mode shapes into measured mode shapes. After calculating \mathbf{T}_{SEREP} , transformation matrix is applied to unmeasured DOFs of the model and measured DOFs for damaged structure are approximated as

$$\boldsymbol{\varphi}_s = \boldsymbol{\varphi}_s^a \mathbf{T}_{SEREP} \quad (3)$$

where $\boldsymbol{\varphi}_s$ is a vector consisting of unmeasured mode shapes. $\boldsymbol{\varphi}_s^a$ is corresponding mode shapes of the model at unmeasured DOFs.

In dynamic expansion method there are two sets of data, called master and slave. Master DOFs are measured and slave are unmeasured DOFs. This method is based on dynamic equilibrium equation written as follows [2]

$$\left(-\omega_{mj}^2 \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} \right) \begin{Bmatrix} \boldsymbol{\varphi}_{mj} \\ \boldsymbol{\varphi}_{sj} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad (4)$$

where $\boldsymbol{\varphi}_{mj}$ and ω_{mj} are measured j th mode shape and natural frequency, respectively. Also, \mathbf{K} and \mathbf{M} are stiffness and mass matrices, respectively. $\boldsymbol{\varphi}_{sj}$ is unmeasured part of j th mode

shape. Rearranging the second equation in Equation (4) gives

$$\boldsymbol{\varphi}_{sj} = - \left(-\omega_{mj}^2 \mathbf{M}_{ss} + \mathbf{K}_{ss} \right)^{-1} \left(-\omega_{mj}^2 \mathbf{M}_{sm} + \mathbf{K}_{sm} \right) \boldsymbol{\varphi}_{mj} \quad (5)$$

In dynamic expansion method, first certain points of the real structure are assumed to be equipped with sensors and the corresponding components of the mode shapes are found. Also, an undamaged structure is modeled by using the finite element method. Subsequently, the stiffness and mass matrices are divided into master (measured) and slave (unmeasured) DOFs as shown in Equation (4). By substituting the measured data from the real structure into Equation (5), unmeasured DOFs are obtained.

3. OPTIMIZATION PROBLEM

In this research the optimization problem can be defined as follows

$$\min_{\boldsymbol{\rho}} : \left\{ f(\boldsymbol{\rho}) = \sum_{i=1}^{nm} \sum_{j=1}^{np} (|\boldsymbol{\varphi}_{ij}| - |\boldsymbol{\varphi}_{ij}^*|)^2 \right\} \quad (6)$$

Subjected to : $0 \leq \rho_k \leq 1$, $k=1, \dots, ne$

where ρ_k is the damage index of the element k , ne , nm and np are the number of elements, considered modes and active degrees of freedom, respectively. $\boldsymbol{\varphi}_{ij}$ and $\boldsymbol{\varphi}_{ij}^*$ are j th component of i th mode shape of the model and real damaged structure, respectively.

For damage detection it is supposed that sensors are installed at some particular points of the structure and mode shapes for those DOFs are measured. The unmeasured DOFs are computed by the aid of modal expansion techniques. Afterwards, a comparison between the model and the damaged data is done and the model is updated to decrease the difference by using the steepest descent method.

In order to parametrize the optimization problem, damages are assumed as density reduction in discretizing finite elements. Therefore, design variables can be defined as element densities ρ_k which are between zero and one. Zero means no material or 100 percent damaged area and one is for solid material with no deficiency. It is noted that any element density between zero and one can be supposed as intensity of the damage. Based on the SIMP topology optimization, any density reduction in element e affects the elasticity matrix \mathbf{C}_e and consequently element matrices as

$$\mathbf{C}_e(\rho_e) = \rho_e^\mu \mathbf{C}_e \quad , \quad \mu \geq 1 \quad (7)$$

$$\mathbf{K}_e(\rho_e) = \rho_e^\mu \int_{\Omega} \mathbf{B}^T \mathbf{C}_e \mathbf{B} d\Omega \quad (8)$$

$$\mathbf{M}_e(\rho_e) = \rho_e^\mu \mathbf{M}_e^* = \int_{\Omega} \rho_e^\mu \mathbf{N}^T d\mathbf{N} d\Omega \quad (9)$$

where μ is penalty factor which is considered with initial amount of 3 and reduces at each step to the final value of 1 that is called the continuation method [35]. \mathbf{B} is the strain-displacement matrix and Ω is the element volume, \mathbf{M}_e^* is mass matrix of undamaged structure, d is density of material and \mathbf{N} is the element shape function matrix.

It is noticed that virtual modes have negative effect on topology optimization in damage detection process. In elements with density of less than 0.1 these virtual modes may occur and the modification shown below needs to be considered to prevent these modes [37]

$$\mathbf{M}_e(\rho_e) = \begin{cases} \rho_e \mathbf{M}_e^* & \rho_e \geq 0.1 \\ (6 \times 10^5 \rho_e^6 - 5 \times 10^6 \rho_e^7) \mathbf{M}_e^* & \rho_e \leq 0.1 \end{cases} \quad (10)$$

Also, mode shapes and natural frequencies are calculated by the modified element matrices and solving the Eigenvalue problem,

$$(-\omega^2 \mathbf{M} \ddot{\mathbf{y}} + \mathbf{K} \mathbf{y}) = 0 \quad (11)$$

$$|\mathbf{K} - \omega^2 \mathbf{M}| = 0. \quad (12)$$

4. SENSITIVITY ANALYSIS

Both numerical and analytical methods might be considered for sensitivity analysis. In numerical sensitivity analysis, finite difference method is used to obtain derivatives of the objective function with respect to design variables. In this approach the structure must be analyzed for all design variables in each iteration which is time consuming. In this research, the derivatives are derived analytically as will be discussed in the following.

To obtain derivative of eigenvalues with respect to the design variable ρ , Equation (12) can be differentiated as

$$\left(\frac{\partial \mathbf{K}}{\partial \rho} - \lambda_i \frac{\partial \mathbf{M}}{\partial \rho} \right) \boldsymbol{\varphi}_i - \frac{\partial \lambda_i}{\partial \rho} \mathbf{M} \boldsymbol{\varphi}_i + (\mathbf{K} - \lambda_i \mathbf{M}) \frac{\partial \boldsymbol{\varphi}_i}{\partial \rho} = 0 \quad (13)$$

where the third term is zero based on Equation (12). Pre-multiplying both sides by $\boldsymbol{\varphi}_i^T$ and using mass-orthogonal eigenvectors ($\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i = 1$), eigenvalue derivative with respect to ρ is given by [38]

$$\frac{\partial \lambda_i}{\partial \rho} = \boldsymbol{\varphi}_i^T \left(\frac{\partial \mathbf{K}}{\partial \rho} - \lambda_i \frac{\partial \mathbf{M}}{\partial \rho} \right) \boldsymbol{\varphi}_i \quad (14)$$

Therefore, derivative of natural frequency is obtained as follows

$$\frac{F(\rho)}{\partial \rho} = \sum_{i=1}^{nm} 2 \left(\frac{\partial \lambda_i}{\partial \rho} \right) \left(\frac{\lambda_i(\rho) - \lambda_i^*}{\lambda_i^{*2}} \right). \quad (15)$$

Derivatives of eigenvectors are also obtained by differentiating Equation (12)

$$\mathbf{F}_i \frac{\partial \boldsymbol{\varphi}_i}{\partial \rho} = - \frac{\partial \mathbf{F}_i}{\partial \rho} \boldsymbol{\varphi}_i \quad (16)$$

where $\mathbf{F}_i = \mathbf{K} - \lambda_i \mathbf{M}$. Afterwards, by differentiating both sides of $\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i = 1$ equation below is obtained

$$2 \boldsymbol{\varphi}_i^T \mathbf{M} \frac{\partial \boldsymbol{\varphi}_i}{\partial \rho} = - \boldsymbol{\varphi}_i^T \frac{\partial \mathbf{M}}{\partial \rho} \boldsymbol{\varphi}_i \quad (17)$$

Since \mathbf{F}_i is singular, Equation (17) which is linearly independent, must be added to the Equation (16) as [26]

$$\begin{bmatrix} \mathbf{F}_i \\ 2 \boldsymbol{\varphi}_i^T \mathbf{M} \end{bmatrix} \frac{\partial \boldsymbol{\varphi}_i}{\partial \rho} = - \begin{bmatrix} \frac{\partial \mathbf{F}_i}{\partial \rho} \\ \boldsymbol{\varphi}_i^T (\frac{\partial \mathbf{M}}{\partial \rho}) \end{bmatrix} \boldsymbol{\varphi}_i \quad (18)$$

$$\frac{\partial \mathbf{F}_i}{\partial \rho} = \left[\frac{\partial \mathbf{K}}{\partial \rho} - \lambda_i \frac{\partial \mathbf{M}}{\partial \rho} - \frac{\partial \lambda_i}{\partial \rho} \mathbf{M} \right] \quad (19)$$

Finally, by rearranging above equations derivatives of objective function with respect to design variables are obtained by the following equation

$$\frac{F(\rho)}{\partial \rho} = \sum_{i=1}^{nm} \sum_{j=1}^{np} 2 \left(\frac{\partial \boldsymbol{\varphi}(\rho)}{\partial \rho} \right) \left(\frac{\boldsymbol{\varphi}_{ij}(\rho)}{|\boldsymbol{\varphi}_{ij}(\rho)|} \right) \left(|\boldsymbol{\varphi}_{ij}(\rho)| - |\boldsymbol{\varphi}_{ij}^*| \right) \quad (20)$$

Derivatives of eigenvectors using Equation (18) are accurate but time consuming, it is easier and faster to obtain derivatives by using Equation (16), but singularity of \mathbf{F}_i is the only problem. A solution to this issue is to solve the inverse problem numerically by minimum residuals method. Thus final formulation for derivatives of eigenvectors is written as below

$$\frac{\partial \boldsymbol{\varphi}_i}{\partial \rho} = -(\mathbf{K} - \lambda_i \mathbf{M})^{-1} \frac{\partial \mathbf{F}_i}{\partial \rho} \boldsymbol{\varphi}_i \quad (21)$$

5. NUMERICAL EXAMPLES

In this section four examples are presented to demonstrate capability of the implemented method. The examples illustrate performance of modal expansion techniques and compare results between fully and partly observed systems. Also, damage detection process is performed based on the proposed topology optimization method for three dimensional continuum structures. Mode shape based objective function is used for all examples. Modulus of elasticity is taken as 2 Pa and Poisson’s ratio is 0.3. Density of materials are assumed to be 0.00785 kg/cm³. Continuation method is used for all examples so that the penalty factor is started from 3 and decreased to 1 after certain iterations.

Example 1. A cantilever beam with 25 cm length, 15 cm width and 25 cm height is considered. Beam is discretized into 5×3×3 elements. Two damaged elements are considered inside the beam with intensity of 50%. First five modes are considered for damage detection process and 300 iterations are chosen for maximum number of iterations. In the first 160 iterations the continuation method is used and the penalty factor is 1 in remaining iterations.

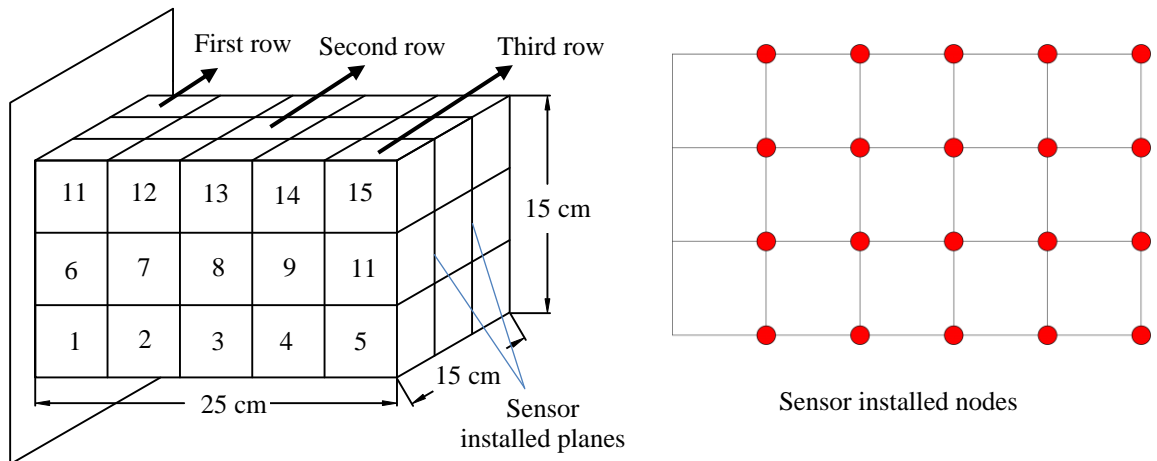
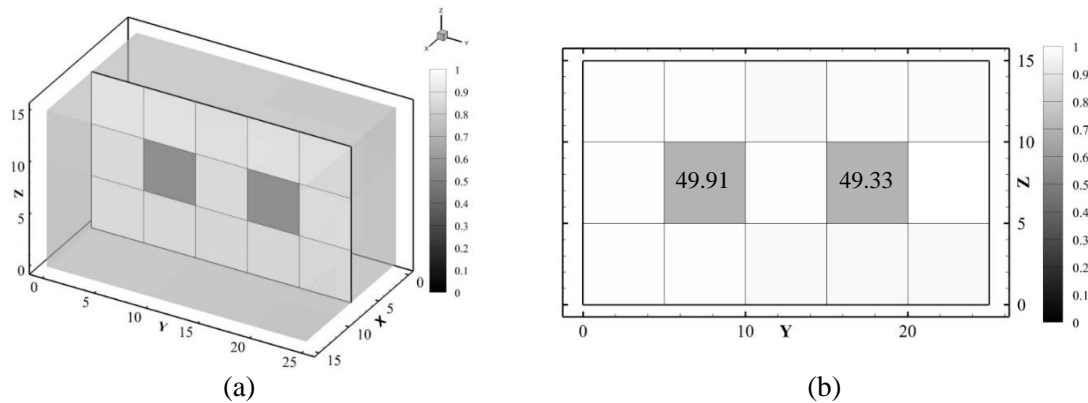


Figure 1. Element numbering in example 1 and sensor placement

Three scenarios are considered for solving the problem as follows. First, sensors are assumed to be at all DOFs. In the second scenario, sensors are installed on nodes at intersection surfaces as shown in Fig. 1.



(a)

(b)

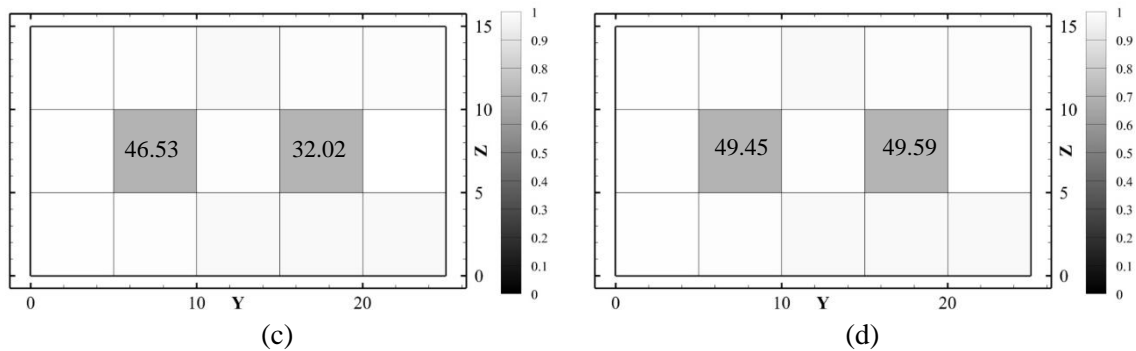


Figure 2. (a) Assumed damaged elements with intensity of 50% (b) results for scenario 1 (all nodes are equipped with sensors) (c) results for scenario 2 (SERP method is used) (d) results for scenario 3 (dynamic expansion method is used)

As it is seen from the results, proposed method can detect damage with considerable accuracy. In scenario 2, the intensities are found rather inaccurate but damage has been localized, precisely. As it is mentioned before, SEREP method is a linear transformation that can calculate unmeasured modes by using measured modes and it is not expected to be very accurate. Instead, as it is seen in scenario 3, dynamic expansion is more precise because it uses more dynamic characteristics than SEREP method. In Fig. 3, iteration histories are illustrated for two damaged elements. It is clear that after certain iterations, damage curve is tending to value of 0.5 which is quantity of the assumed damage.

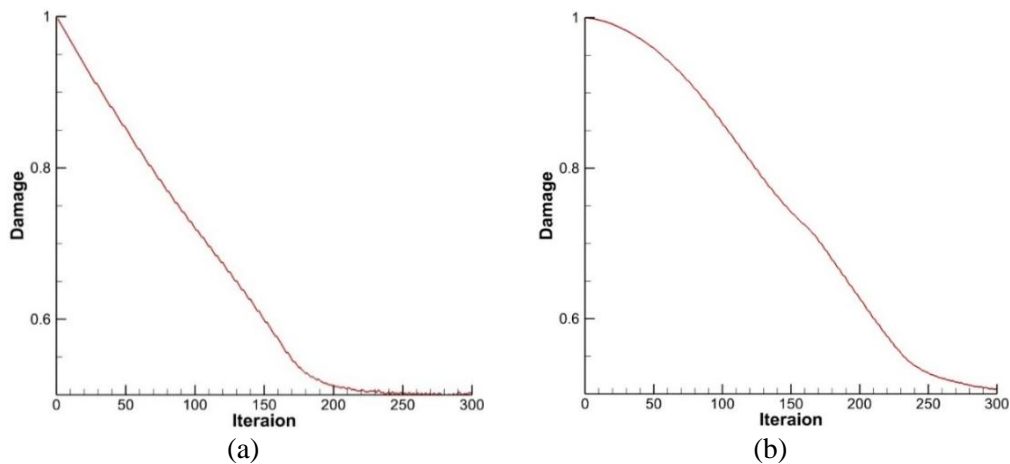


Figure 3. Iteration history for intensity of damage in (a) element number 7 in second row and (b) element number 9 in second row

Example 2. A beam with the same dimensions as example 1 is considered with different boundary conditions as shown in Fig. 4(a). The beam is discretized into $10 \times 6 \times 6$ finite elements. Different damage intensities are assumed in 12 elements as shown in Fig. 4(b). First five mode shapes are considered for damage detection process. Two scenarios are taken for the number of sensors which are installed (1) at all DOFs and (2) at nodes in all intersection surfaces as shown in Fig. 4(c) which consist 385 DOFs out of 539 DOFs.

Dynamic expansion method is used to approximate unmeasured DOFs. Results for both scenarios are shown in Fig. 5(a). The objective function history is also shown in Fig. 5(b).

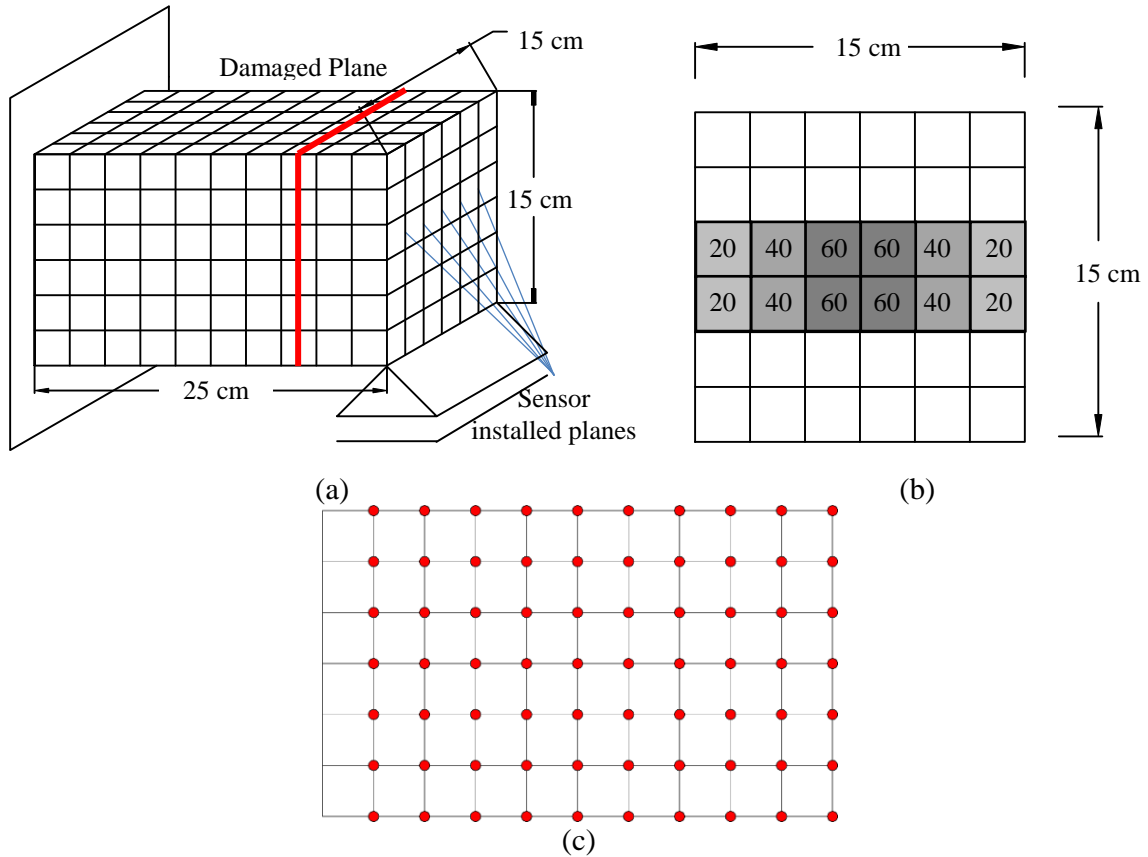


Figure 4. Problem definition for Example 2

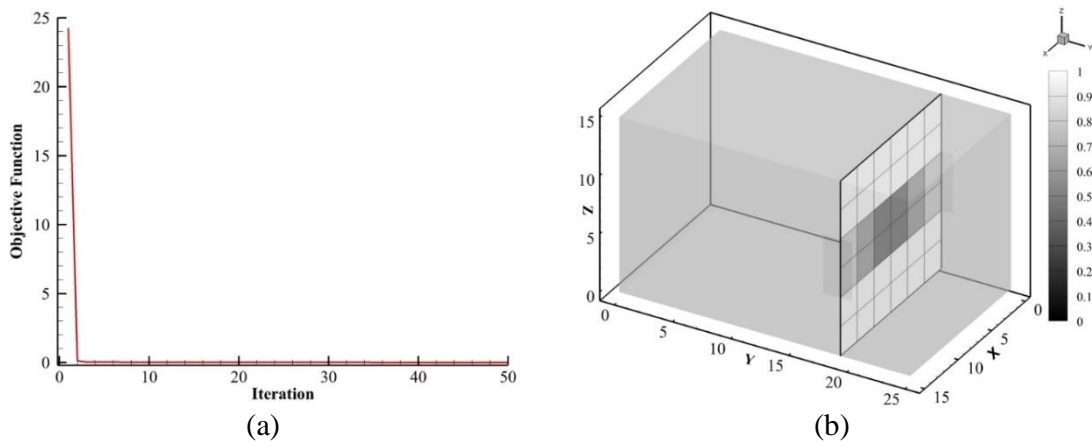


Figure 5. Damage location and iteration history for Example 2.

As shown in Fig. 5(b), damage location is precisely determined. Fig. 5(a) shows iteration history of the objective function. The horizontal part of objective function curve declares

convergence of the algorithm after a few iterations. Fig. 6(a) shows damage intensities when all DOFs have sensors. It is observed that both locations and intensities are derived accurately by the algorithm. Fig. 6(b) is the result of damage detection by using dynamic expansion. Obtained intensities are not as precise as 6(a) as the number of sensors are decreased.

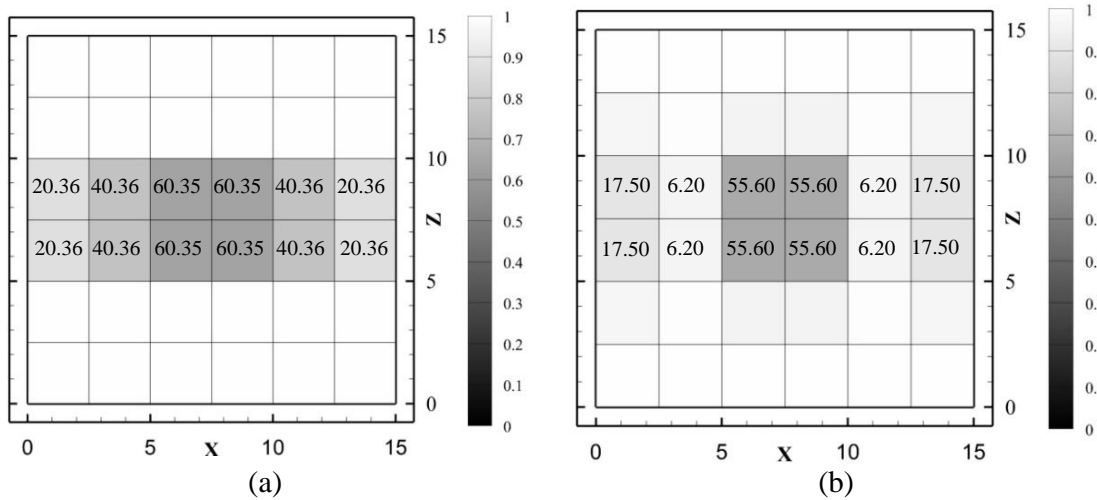


Figure 6. Damage intensities when sensors are in (a) all DOFs (b) intersection planes.

Example 3. A long rectangular beam with dimensions of 60 cm length, 4 cm width and 4 cm height is considered. Beam is discretized into $40 \times 4 \times 4$ with a total of 640 finite elements. Eight elements, at the middle of the beam are assumed to be damaged with intensity of 50 percent. Location and intensity of damages and boundary conditions are shown in Fig. 7. First five modes are considered to define the objective function for damage detection optimization process. Sensor arrangement for this example is also shown in Fig. 8.

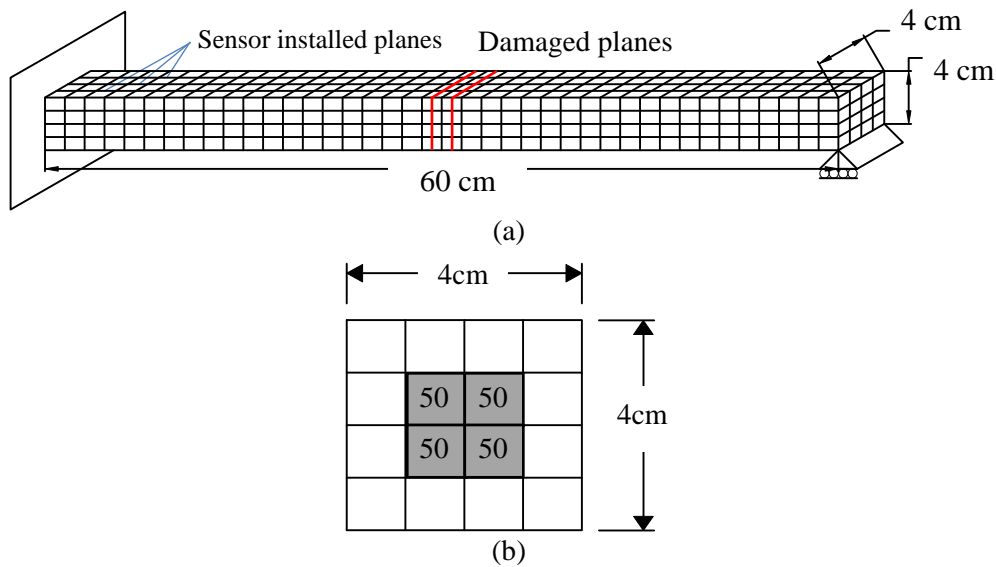


Figure 7. (a) Beam model and boundary conditions (b) damage scenario

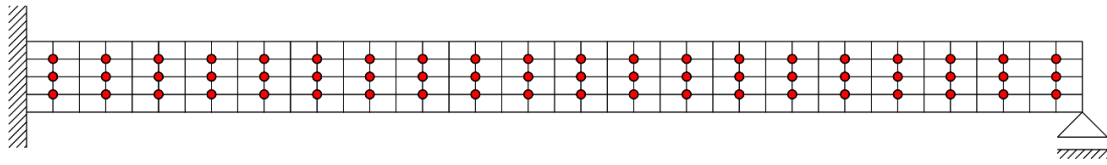


Figure 8. Sensor arrangement in example 3

This example is solved for two scenarios. In the first scenario, all nodes are equipped with sensors. In the second scenario, nodes shown in Fig. 8 are equipped with sensors and dynamic expansion method is used to approximate the unmeasured modal data. Table 1 shows all three scenarios in terms of number of sensors.

Table 1: Number of sensors in scenarios

Scenario	Number of nodes	Number of Sensors
1	1025	1025
2	1025	180

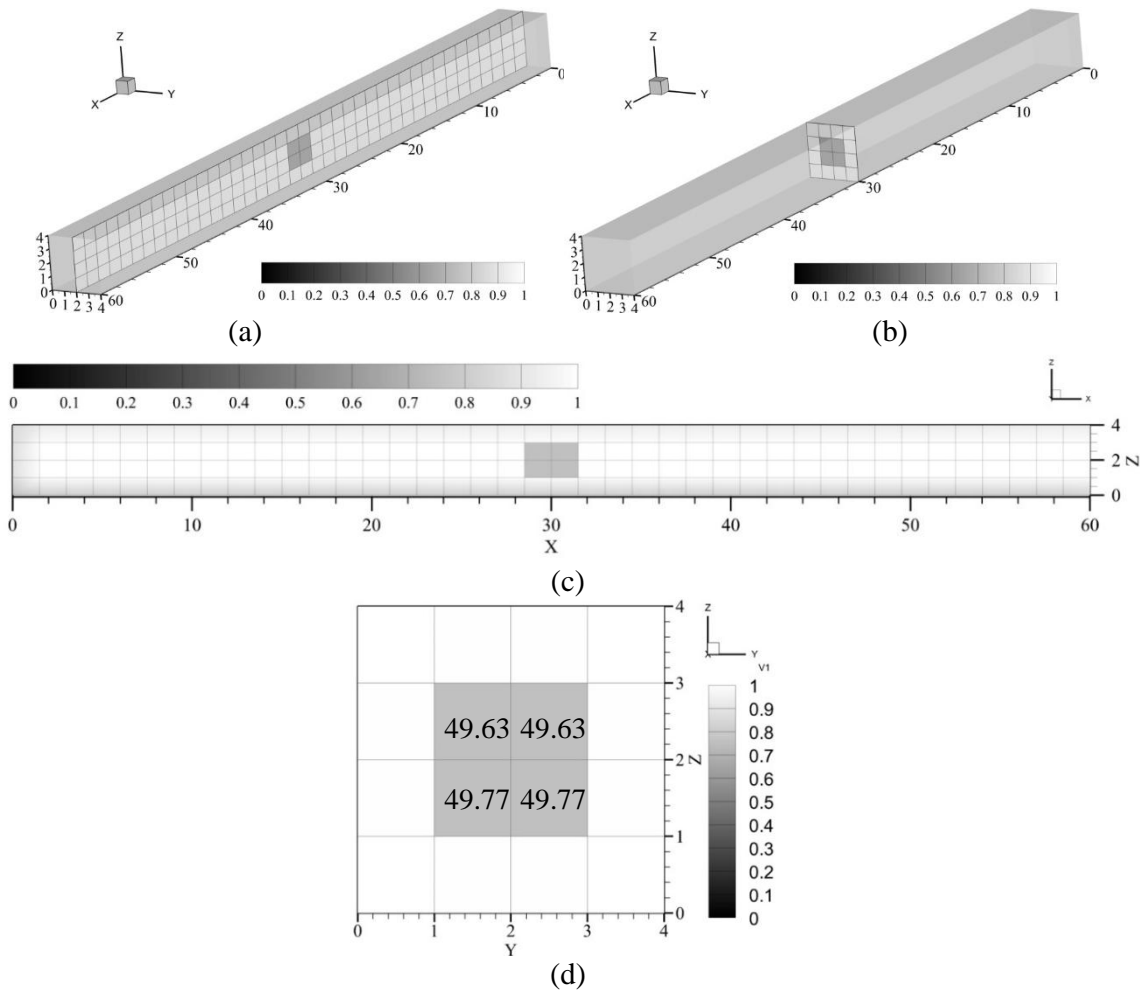


Figure 9. Results for scenario 1, (a, b) 3d Views (c) longitudinal and (d) lateral cross section

Results for first scenario is shown in Fig. 9 that shows accuracy of optimization algorithm in three dimensional problems. Location and intensity of damages are detected properly with reasonable accuracy. The iteration history for objective function and the average intensity of damaged elements are shown in Fig. 10.

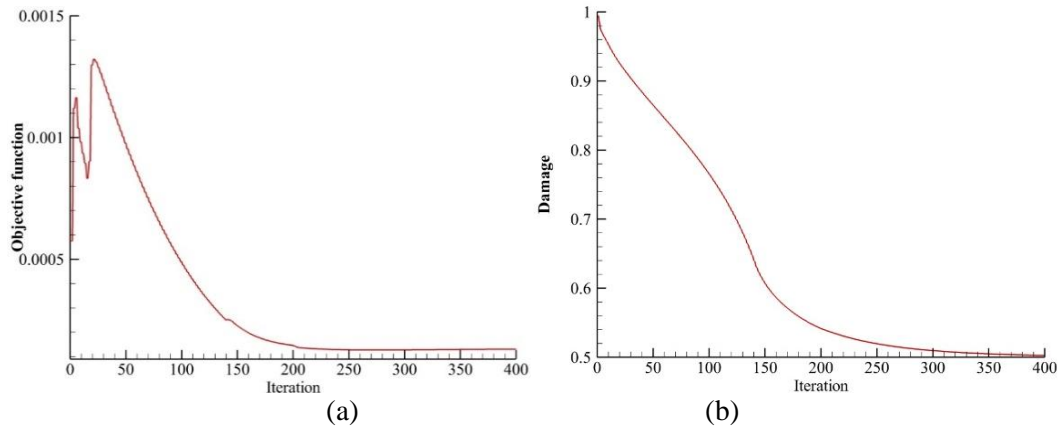
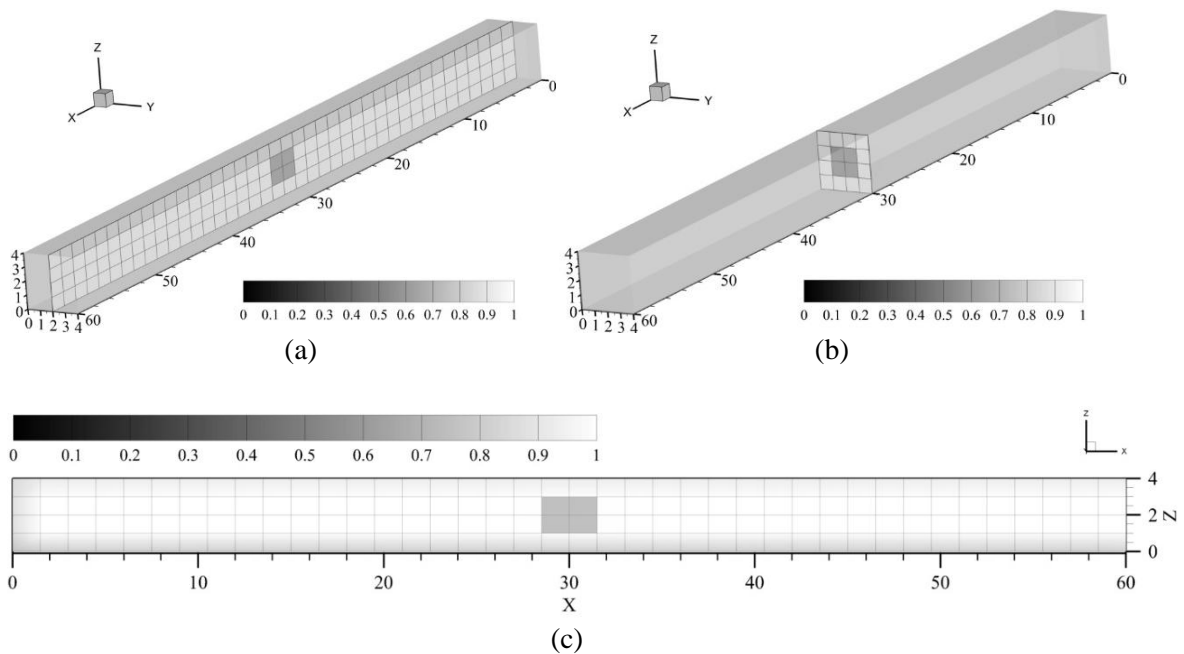


Figure 10. Iteration history of (a) objective function and (b) average intensity of damaged elements

The result for the second scenario is depicted in Fig. 11. Fig. 12 also shows the iteration histories of objective function and average intensity of damaged elements. Results show that the proposed method has detected damage location and its quantity with reasonable accuracy and the optimization process converges.



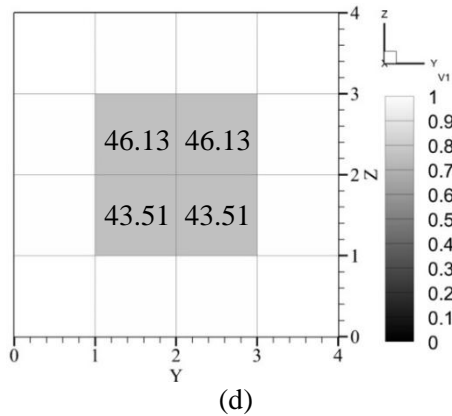


Figure 11. Results for scenario 2 (a, b) 3d views (c) longitudinal and (d) lateral cross section

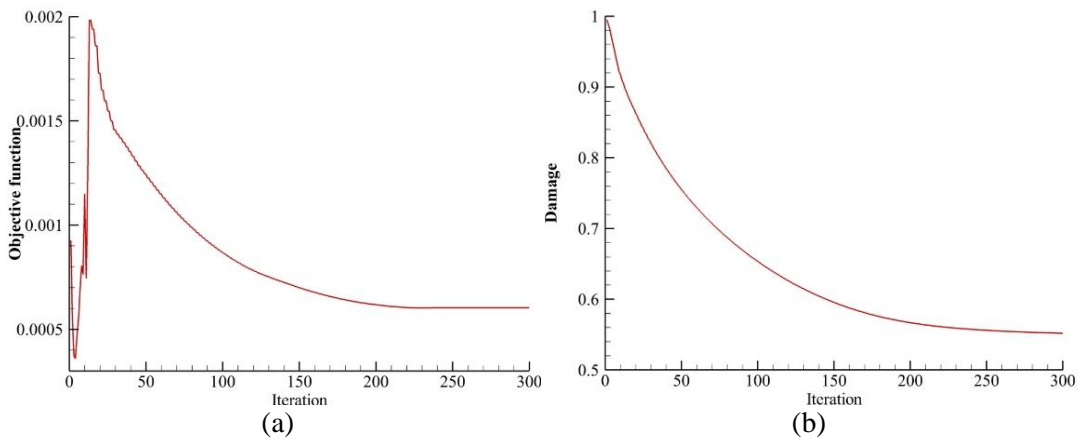


Figure 12. Iteration history of (a) objective function and (b) average intensity of damaged elements

For the sake of comparison, results for both scenarios are demonstrated in Table 2. As it is expected more error occurred in the second scenario as the number of input data is fewer than the first scenario.

Table 2: Results of scenarios

Scenario	Assigned damage	Detected damage	Error
1	50%	49.70%	0.6%
2	50%	44.82%	10.36%

Example 4. An I-shape beam with length of 60 cm is considered for this example. Cross section of the beam as well as boundary conditions are shown in Fig. 13. The beam is discretized into 800 finite elements. First five mode shapes are considered for defining the objective function and the continuation method is used for the first 200 iterations. Since damage is assumed in the web, sensors are considered to be on both sides of the web. The assumed damage is also shown in Fig. 13(c) and sensor arrangement is shown in Fig. 14.

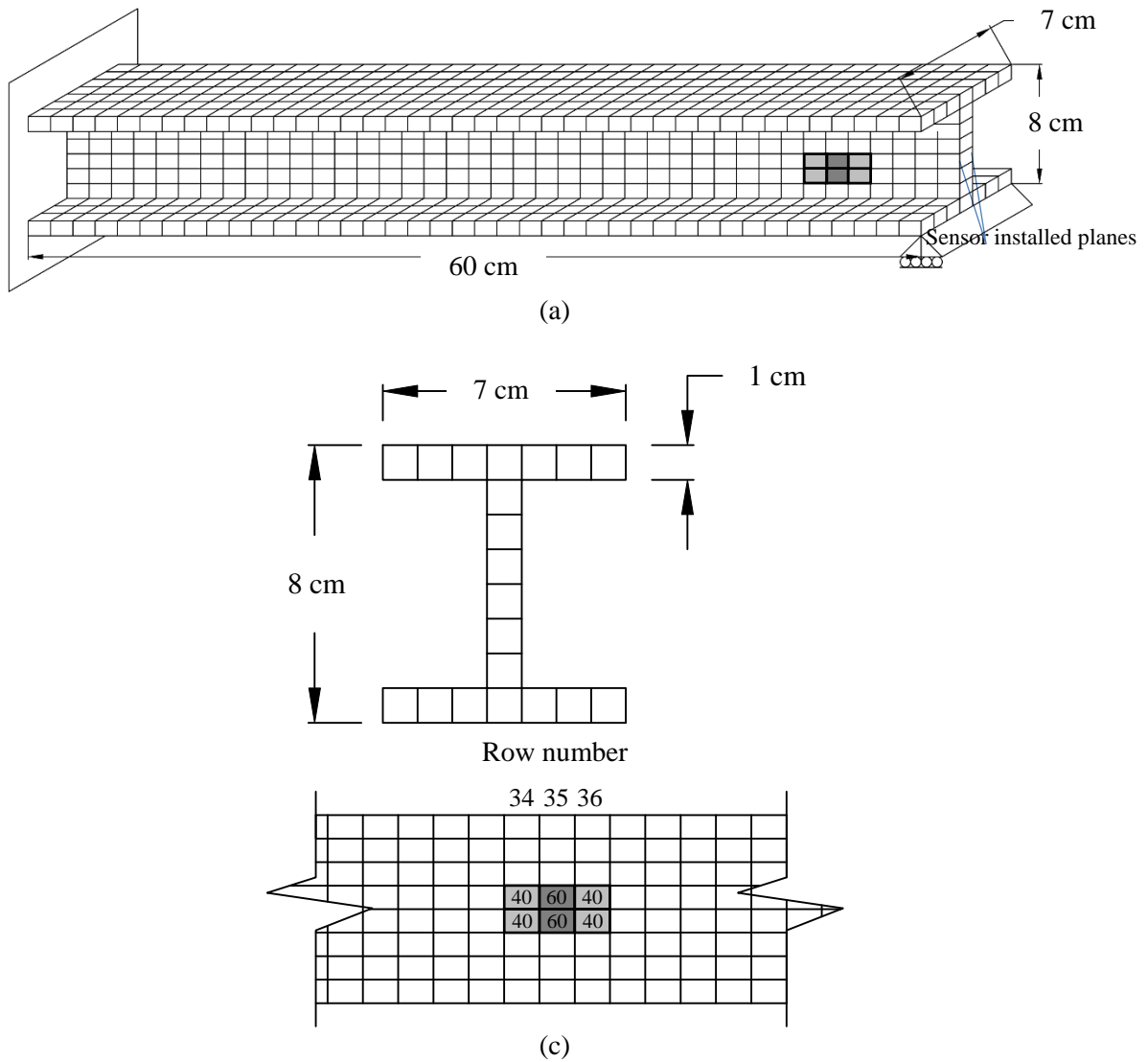


Figure 13. (a) Boundary conditions (b) beam section (c) damage scenario for example 4

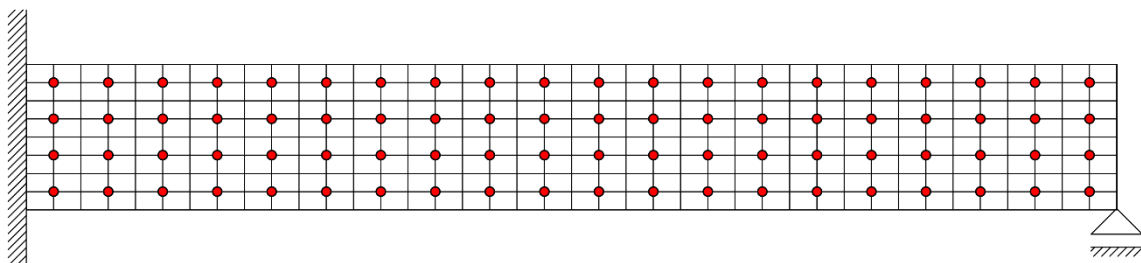


Figure 14. Sensor arrangements used in example 4

Damage detection process is applied to this example and the proposed method could accurately obtain the damage location and damage intensity. First, modal data for all 1722

nodes are assumed to be available. Second scenario is more challenging and fewer nodes are equipped with sensors and less data is available. As it is seen in Fig. 14, 160 nodes on both sides of the web are considered. Table 3 shows details of number of sensors. Results for first scenario are shown in Fig. 15. No additional damage is detected and damage intensities are precisely computed.

Table 3: Number of sensors in scenarios

Scenario	Number of nodes	Number of sensors
1	1722	1722
2	1722	160

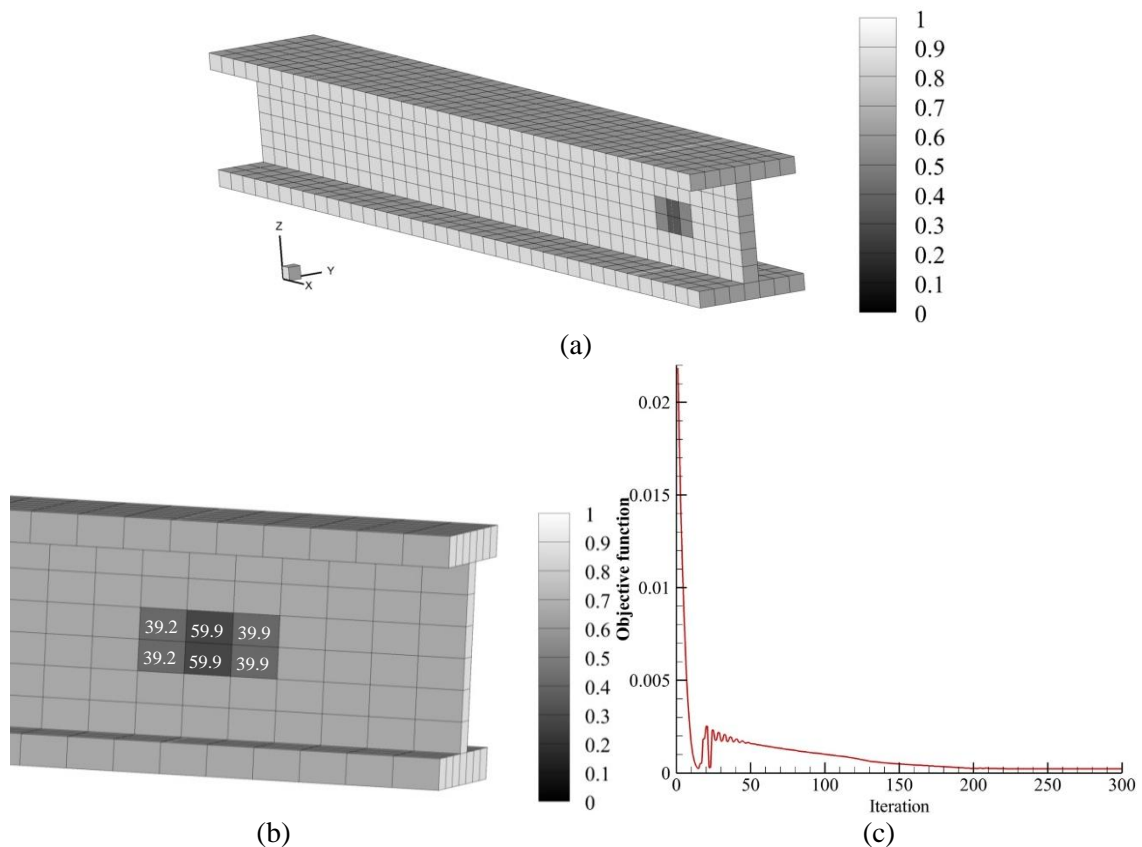


Figure 15. (a) 3D model. (b) Detected damages. (c) Iteration history

As it is mentioned before, in the second scenario, number of sensors are less than first scenario. After performing optimization process, the results are shown in Fig. 16. Damage locations are detected precisely and intensities are obtained with some errors. As it is seen, some extra damages are detected around the true damage. For the sake of comparison, Table 4 shows both scenarios along with damage intensities.

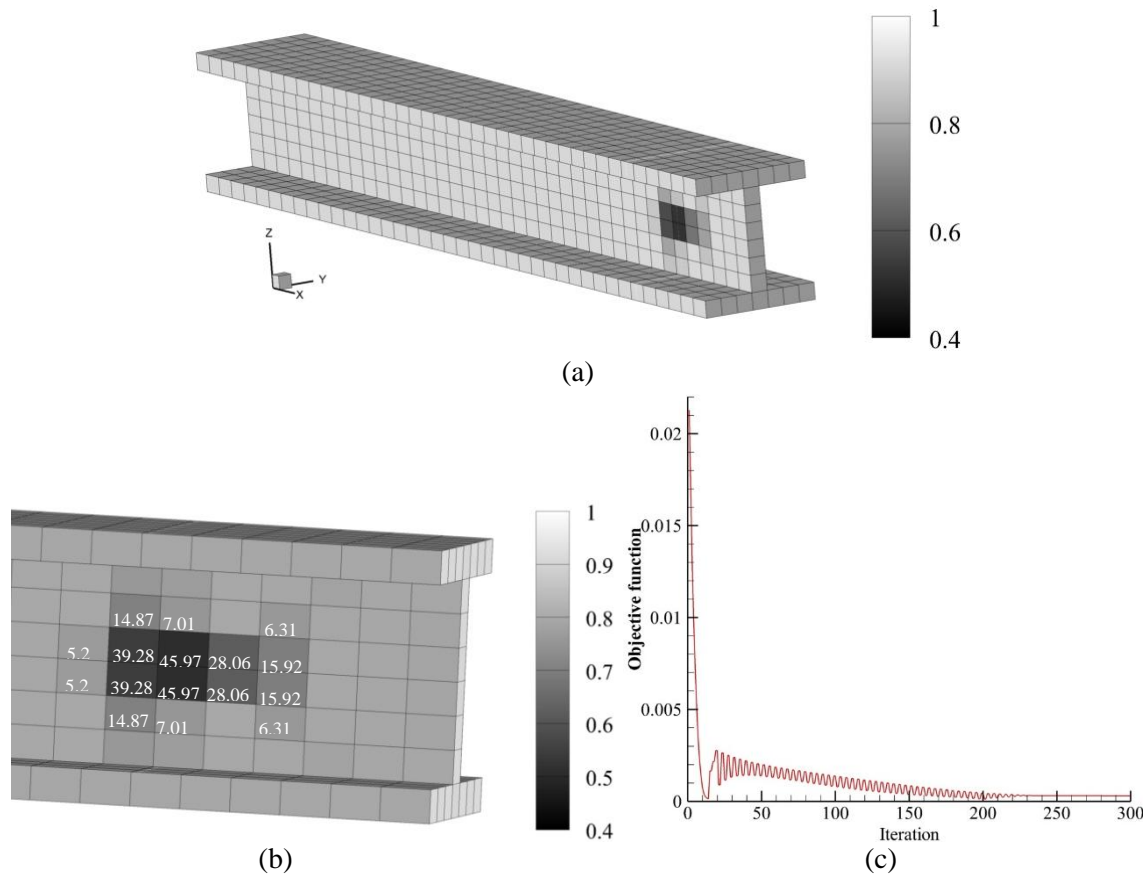


Figure 16. (a) 3D model (b) detected damages (c) iteration history.

Table 4: Results of both scenarios in example 4

Scenario	Assigned damage	Detected damage	Error
1	40%	39.61%	0.9%
	60%	59.97%	0.05%
2	40%	33.655%	15.86%
	60%	45.97%	23.38%

6. CONCLUSION

In this paper, mode expansion techniques are used to obtain mode shapes at all DOFs by using measured modal data in three dimensional structures. In the next step, topology optimization is utilized to minimize the difference between obtained mode shapes of real structure and its numerical model to find damages and their intensities. Analytical sensitivity analysis is achieved and the optimization problem is solved by the steepest descent method. Damages are modeled as density reduction in the model and the SIMP method is used for topology parameterization. Numerical examples show that proposed method can identify damages with reasonable precision in terms of both location and severity. However, due to

approximating the unmeasured data, accuracy of results is dependent on the number and arrangement of sensors especially in identifying damage intensities.

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