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A NEW APPROACH FOR SOLVING FULLY FUZZY QUADRATIC PROGRAMMING PROBLEMS

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ABSTRACT

Quadratic programming (QP) is an optimization problem wherein one minimizes (or maximizes) a quadratic function of a finite number of decision variable subject to a finite number of linear inequality and/ or equality constraints. In this paper, a quadratic programming problem (FFQP) is considered in which all cost coefficients, constraints coefficients, and right hand side are characterized by L-R fuzzy numbers. The FFQP problem is converted into the fully fuzzy linear programming using the Taylor series and hence into a linear programming problem which may be solved by applying GAMS Software. Finally, an example is given to illustrate the practically and the efficiency of the method.

Keywords: Fully Fuzzy Quadratic; $L-R$ Fuzzy Numbers; Taylor Series; Fully Fuzzy Linear Programming; Ranking Function; Linear Programming; Fuzzy Optimal Solution.

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1. INTRODUCTION

Quadratic programming (QP) is an optimization problem whose objective function is quadratic function and the constraints are linear equalities or inequalities. QP is widely used in real world problem to optimize portfolio selection problem, in the regression to perform the least square method, in chemical plants to control scheduling, in the sequential quadratic programming, economics, engineering design etc. Since the QP is the most interesting class of the optimization, so it is known as the NP- hard. There are several method and algorithms for solving the QP problem introduced by Pardalos and Rosen [1], Horst and Tuy [2] and Bazaraa et al. [3]. Beck and Teboulle [4] established a necessary global optimality condition for the nonconvex QP optimization problem with binary constraints. Kochenberger *et al*. [5] studied the unconstrained binary quadratic programming problem. Xia [6] explored local optimality conditions for obtaining new sufficient optimality conditions for nonconvex quadratic optimization with binary constraints. Bonami *et al.* [7] introduced an effective linear programming based a computational techniques for solving non-convex quadratic programs with box constraints. Abbasi [8] developed a new model for solving convex QP problems based on differential-algebraic equations. Pramanik and Dey [9] proposed apriority based fuzzy programming approach for solving multi- objective QP problem.

In spite of having a vast decision making experience, the decision maker cannot always articulate the goals precisely. Decision-making in a fuzzy environment, developed by Bellman and Zadeh [10] improved and a great help in the management decision problems. Based on Zadeh' extension principle [11] the fuzzy QP is transformed into a pair of two-level mathematical programs by evaluating the upper and lower bounds of the objective value at possibility level. Zimmermann [12] proposed the fuzzy set theory and its applications. Zimmermann [13] proposed the fuzzy programming with several objective functions, fuzzy sets and systems. Nasseri [14] defined QP with fuzzy numbers where trapezoidal and / or triangular fuzzy numbers characterizes the cost coefficients, constraints coefficients, and right hand side. Kheirfam [15] used a fuzzy ranking and arithmetic operations to transform the QP problem with fuzzy numbers in the coefficients and variables into the corresponding deterministic one and solved it to obtain a fuzzy optimal solution. Allahviranloo and Moazam [16] introduced a new concept of second power of fuzzy numbers that is provide an analytical and approximate solution for fully fuzzy quadratic equations. Taghi- Nezad and Taleshian [17] proposed a solution method for solving a special class of fuzzy QP problems with fuzziness in relations. Gao and Ruan [18] presented a canonical duality theory for solving quadratic minimization problems subject to either box or integer constraints. Sun *et al*. [19] investigated the duality gap between the binary quadratic optimization problem and its semidefinite programming relaxation. Gill and Wong [20] proposed an active set method for solving a genetic QP problem with both equality and inequality constraints. Syaripuddin *et al.* [21] applied two- level programming approach for solving interval valued variables quadratic programming by dividing the problem into two problems the best optimum and the worst problems. Takapoui *et al.* [22] proposed an algorithm for approximately minimizing convex quadratic function over the intersection of affine and separable constraints. Shi *et al.* [23] proposed an effective algorithm for solving quadratic programming problem with quadratic constraints globally. Gabr [24] presented a comprehensive methodology for solving and analyzing quadratic and nonlinear programming in fully fuzzy environment. Mirmohesni and Nasseri [25] introduced QP problem with triangular fuzzy in all of objective function coefficients and the fuzzy right-hand side of the constraints and proposed a new approach to derive the fuzzy objective value for the problem. Maheswari [26] proposed a new approach based on Kuhn- Tucker conditions for solving fuzzy QP problems.

In this paper, FFQP problem is studied. Proposed approach is applied for obtaining the optimal compromise solution for the problem without converting it into the corresponding deterministic problem.

The outlay of the paper is organized as follows: Section2 presented some preliminaries related to the $L - R$ fuzzy numbers and their arithmetic operations. Section3 formulates fully fuzzy quadratic programming problem. In Section4, a proposed approach is applied for solving the problem introduced in Section2. In Section 5, a numerical example to illustrate the efficiency of the solution approach is given. Finally, some concluding remarks are reported in Section 6.

2. PRELIMINARIES

This section introduces some of basic concepts and results related to $L-R$ fuzzy numbers and their arithmetic operations.

Definition 1[27]. A triangular fuzzy number can be represented completely by a triplet $\tilde{A} = (a, b, c)$, and has membership

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \le x \le b, \\ \frac{c-x}{c-b}, & b \le x \le c, \\ 0, & x > c. \end{cases} \quad (1)$$

Definition 2. The ordinary representative of the fuzzy number $A = (a, b, c)$ is given by $\hat{A} = \frac{a+2b+c}{4}$.

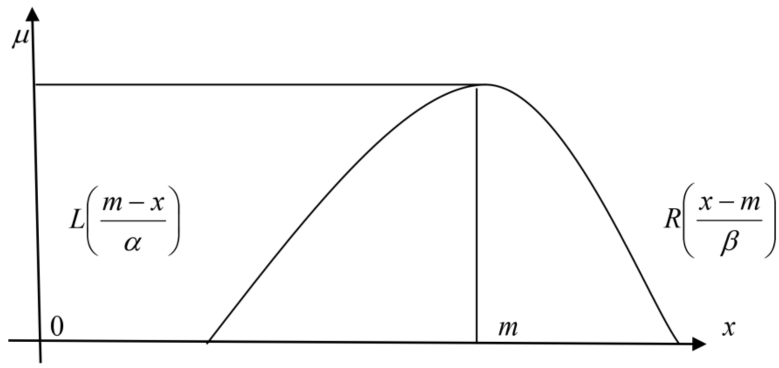

Figure 1. *L-R* Fuzzy number representation.

Definition 3 [28]. A fuzzy number $\tilde{A} = (x, \alpha, \beta)_{LR}$ is said to be an $L-R$ fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \le m, \alpha > 0 \\ R\left(\frac{x-m}{\beta}\right), x \ge m, \beta > 0 \end{cases} \quad (2)$$

where m is the mean value of $\tilde{A}$ and α and β are left and right spreads, respectively, and a function $L(.)$ is a left shape function satisfying: $L(x) = L(-x), L(0) = 1, L(x)$ is nonincreasing on $[0, \infty[$.

It is noted that a right shape function $R(.)$ is similarly defined as $L(.)$.

For two $L-R$ fuzzy numbers $\tilde{A} = (x, \alpha, \beta)_{LR}$,and $\tilde{B} = (y, \gamma, \sigma)_{LR}$, the arithmetic operations are:

1. Addition: $\widetilde{A} \oplus \widetilde{B}$

$$(x, \alpha, \beta)_{LR} \oplus (y, \gamma, \sigma)_{LR} = (x + y, \alpha + \gamma, \beta + \sigma)_{LR} \tag{3}$$

2. Opposite: $-\widetilde{A}$

$$-(x, \alpha, \beta)_{LR} = (-x, \beta, \alpha)_{LR} \tag{4}$$

3. Subtraction: $\widetilde{A} \ominus \widetilde{B}$

$$(x, \alpha, \beta)_{LR} \ominus (y, \gamma, \sigma)_{RL} = (x - y, \alpha + \sigma, \beta + \gamma)_{LR} \tag{5}$$

4. Multiplication: $\widetilde{A} \odot \widetilde{B}$

$$\widetilde{A} \odot \widetilde{B} = \begin{cases} \text{If } \widetilde{A} \succ 0, \widetilde{B} \succ 0, \text{then } (x, \alpha, \beta)_{LR} \odot (y, \gamma, \sigma)_{LR} \cong (xy, x\gamma + y\alpha, x\sigma + y\beta)_{LR}, \\ \text{If } \widetilde{A} \prec 0, \widetilde{B} \succ 0, \text{then } (x, \alpha, \beta)_{LR} \odot (y, \gamma, \sigma)_{LR} \cong (xy, y\alpha - x\sigma, y\beta - x\sigma)_{RL}, \\ \text{If } \widetilde{A} \prec 0, \widetilde{B} \prec 0, \text{then } (x, \alpha, \beta)_{LR} \odot (y, \gamma, \sigma)_{LR} \cong (xy, -y\beta - x\sigma, -y\sigma - x\gamma)_{RL.} \end{cases} \tag{6}$$

5. Scalar multiplication: $\lambda \odot \tilde{A}$

$$\lambda \odot (x, \alpha, \beta)_{LR} = \begin{cases} (\lambda x, \lambda\alpha, \lambda\beta)_{LR}, & \lambda \succ 0 \\ (\lambda x, -\lambda\beta, -\lambda\alpha)_{RL}, & \lambda \prec 0 \end{cases} \tag{7}$$

3. PROBLEM FORMULATION AND SOLUTION CONCEPTS

A fully fuzzy quadratic programming problem is formulated in matrix form as follows

$$\min \tilde{Z} = \tilde{C}^{T} \odot X \oplus \frac{1}{2} X^{T} \odot \tilde{Q} \odot X \tag{8}$$

Subject to:

$$\widetilde{A} \odot X \begin{pmatrix} \precsim \\ \approx \end{pmatrix} \tilde{b}, X \geq 0. \tag{9}$$

Problem (8)-(9), can be rewritten in the following compact form as

$$\min \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j \odot x_j \oplus \frac{1}{2} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \odot \tilde{q}_{ij} \odot x_j \right) \tag{10}$$

Subject to:

$$x \in \widetilde{M} = \left\{ \begin{array}{c} \sum_{j=1}^{n} \tilde{a}_{ij} \odot x_j \preccurlyeq \tilde{b}_i \, , i = 1, 2, \dots, m_1, \sum_{j=1}^{n} \tilde{a}_{ij} \odot x_j \approx \tilde{b}_i \, , i = m_{i+1}, \dots, m \\ x_j \geq, j = 1, 2, \dots, n \end{array} \right\} \quad (11)$$

where, $\tilde{C} = (\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_n)$, and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m)$ are fuzzy cost vector and fuzzy right-hand side vector. $X = (x_1, x_2, \dots, x_n)$ is a vector of variables, and also $\tilde{Q} = [q_{ij}]_{n\times n}$ is a matrix of quadratic form which is symmetric and positive semi definite, and $\widetilde{A} = [\tilde{a}_{ij}]_{m\times n}$. It is assumed that all of $\widetilde{A}$, $\tilde{b}$ $\tilde{C}$, and $\widetilde{Q} \in F(R)$, where $F(R)$ denotes the set of all $L - R$ fuzzy numbers.

Definition 4. A non-negative fuzzy vector x_j is said to be fuzzy feasible solution for problem (10)- (11) if it satisfies the constraints (11).

Definition 5. A fuzzy feasible solution x_j^* is called a fuzzy optimal solution for problem (10)- (11), if

$$\left[\sum_{j=1}^{n} \tilde{c}_j \odot x_j^* \oplus \frac{1}{2} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_i^* \odot \tilde{q}_{ij} \odot x_j^* \right) \right] (\preccurlyeq) \cdot \left[\sum_{j=1}^{n} \tilde{c}_j \odot x_j \oplus \frac{1}{2} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \odot \tilde{q}_{ij} \odot x_j \right) \right]; \forall\, x_j \quad (12)$$

Let $\tilde{c}_j, \tilde{q}_{ij}, \tilde{a}_{ij}$, and $\tilde{b}_i$ represent by the $L - R$ fuzzy numbers, $(c_j, \alpha_j, \beta_j)_{LR}, (q_{ij}, \delta_{ij}, \varepsilon_{ij})_{LR}, (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR}$, and $(b_i, \vartheta_i, \rho_i)$, erespectively. Then problem (10)- (11) becomes

$$\min \tilde{Z} = \sum_{j=1}^{n} (c_j, \alpha_j, \beta_j)_{LR} \odot x_j \oplus \frac{1}{2} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \odot (q_{ij}, \delta_{ij}, \varepsilon_{ij})_{LR} \odot x_j \right) \quad (13)$$

Subject to:

$$x \in \widetilde{M} = \left\{ \begin{array}{c} \sum_{j=1}^{n} (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR} \odot x_j \preccurlyeq (b_i, \vartheta_i, \rho_i)_{LR} \, , i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} (a_{ij}, \epsilon_{ij,} \theta_{ij})_{LR} \odot x_j \approx (b_i, \vartheta_i, \rho_i)_{LR} \, , i = m_{i+1}, \dots, m \\ x_j \geq, j = 1, 2, \dots, n \end{array} \right\}. \quad (14)$$

Definition 6 [29]. Assume that function G has first order partial derivatives (i. e., is of class $C^{(1)}$). The first two term of the Taylor series generated by $G(x_1, x_2, \dots, x_n)$ at $B(a_1, a_2, \dots, a_n)$ is

$$G(B) + \frac{\partial}{\partial x_1} G(B)(x_1 - a_1) + \frac{\partial}{\partial x_2} G(B)(x_2 - a_2) + \cdots + \frac{\partial}{\partial x_n} G(B)(x_n - a_n) = 0 \qquad (15)$$

4. PROPOSED APPROACH

In this section, the steps of the proposed approach for solving fully fuzzy quadratic programming is illustrated as

Step 1: Consider the fully fuzzy quadratic problem (12)- (13) and choose any arbitrary feasible non- zero point initially, say $\check{x}$,

Step 2: Using the definition6, expand the objective function (12) to the Taylor series at the$\check{x}$, so as to obtain fully fuzzy linear programming (FFLP) problem,

Step 3: Consider the FFLP problem

$$\min \widetilde{W} = \sum_{j=1}^{n} (d_j,\ \varphi_j,\ \omega_j)_{LR} \odot x_j \qquad (16)$$

Subject to:

$$x \in \widetilde{M} = \left\{ \begin{array}{c} \sum_{j=1}^{n} \left(a_{ij}, \epsilon_{ij,} \theta_{ij}\right)_{LR} \odot x_j \preccurlyeq (b_i, \vartheta_i, \rho_i)_{LR}, i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} \left(a_{ij}, \epsilon_{ij,} \theta_{ij}\right)_{LR} \odot x_j \approx (b_i, \vartheta_i, \rho_i)_{LR}\ , i = m_{i+1}, \dots, m \\ x_j \geq, j = 1, 2, \dots, n \end{array} \right\} \qquad (17)$$

Rewrite the problem (16) as in the following form

$$\min \widetilde{W} = \sum_{j=1}^{n} (d_j{}^l,\ d_j{}^c, d_j{}^u) \odot x_j \qquad (18)$$

Subject to:

$$x \in \widetilde{M} = \left\{ \begin{array}{c} \sum_{j=1}^{n} (a_{ij}{}^{l},\ a_{ij}{}^{c}, a_{ij}{}^{u}) \odot x_j \preccurlyeq (b_i{}^{l},\ b_i{}^{c},\ b_i{}^{u}), i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} (a_{ij}{}^{l},\ a_{ij}{}^{c}, a_{ij}{}^{u}) \odot x_j \approx (b_i{}^{l},\ b_i{}^{c},\ b_i{}^{u}), i = m_{i+1}, \dots, m \\ x_j \geq, j = 1, 2, \dots, n \end{array} \right\} \quad (19)$$

Based on the ranking function in definition2, convert the problem (18) into the following linear programming problem as:

$$\min \widetilde{W} = \sum_{j=1}^{n} \frac{d_j{}^{l} + 2\, d_j{}^{c} + \ d_j{}^{u}}{4} x_j \quad (20)$$

Subject to

$$x \in \widetilde{M} = \left\{ \begin{array}{c} \sum_{j=1}^{n} \frac{a_{ij}{}^{l} +\ 2a_{ij}{}^{c} +\ a_{ij}{}^{u}}{4} x_j \leq \frac{b_i{}^{l} + 2\, b_i{}^{c} +\ b_i{}^{u}}{4}, i = 1, 2, \dots, m_1, \\ \sum_{j=1}^{n} \frac{a_{ij}{}^{l} +\ 2a_{ij}{}^{c} +\ a_{ij}{}^{u}}{4} x_j = \frac{b_i{}^{l} + 2\, b_i{}^{c} +\ b_i{}^{u}}{4}, i = m_{i+1}, \dots, m \\ x_j \geq, j = 1, 2, \dots, n \end{array} \right\} \quad (21)$$

Let $\overline{x}$ be the solution of problem (21).

Step 4: Expand the objective function (12) to the Taylor series at the solution$\overline{x}$,

Step 5: Solve the problem resulted from Step 4 subject to the given constraints to obtain another solution $\underline{x}$,

Step 6: If the two solutions $\overline{x}$ and $\underline{x}$ overlap then the solution of the problem (20) is obtained, and then calculate the fuzzy objective value and stop. Otherwise, assign $\underline{x}$ to $\overline{x}$ and return to the Step 4.

The flow chart of the proposed approach can be illustrated as in the Fig. 2 below.

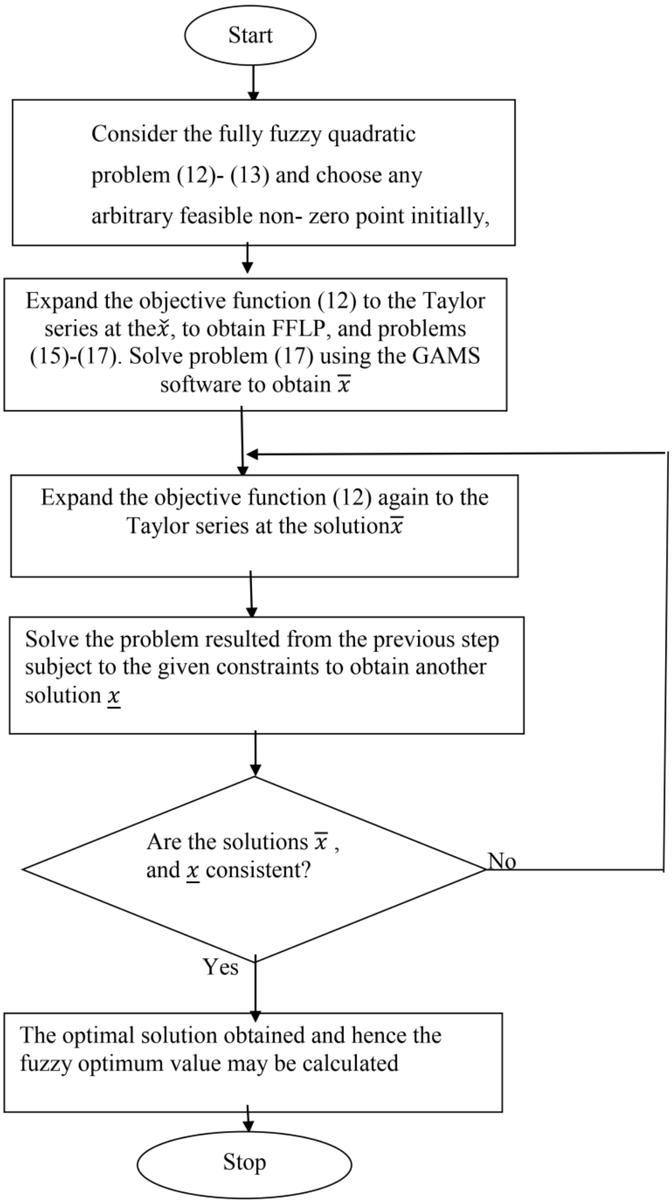

Figure 2. The flow chart of the proposed approach.

5. NUMERICAL EXAMPLE

Consider the FFQP problem as

$$\min \tilde{Z} = \left[\begin{array}{c} (-2,-1,1)_{LR} \odot x_1 \oplus (-6,3,3)_{LR} \odot x_2 \oplus \\ (x_1\ x_2) \begin{pmatrix} (1,0,1)_{LR} & (-2,2,6)_{LR} \\ (-2,-2,6)_{LR} & (3,7,3)_{LR} \end{pmatrix} \otimes \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{array} \right] \qquad (22)$$

Subject to

$$(1,1,1)_{LR} \odot x_1 \oplus (1,2,2)_{LR} \odot x_2 \preccurlyeq (3,5,1)_{LR},$$
$$(-2,2,6)_{LR} \odot x_1 \oplus (2,1,1)_{LR} \odot x_2 \preccurlyeq (3,5,1)_{LR}, \quad (23)$$
$$(2,2,2)_{LR} \odot x_1 \oplus (1,0,0)_{LR} \odot x_2 \preccurlyeq (3,2,2)_{LR}, x_1, x_2 \geq 0.$$

Step 1-2: Choose $(0.5, 1)$as an arbitrary initial feasible non- zero point, and expanding the function to the Taylor series at it as:

$$\frac{\partial \tilde{Z}}{\partial x_1} = (-2,-1,1) \oplus \frac{1}{2}[(2,0,2) \odot x_1 \oplus (-4,4,12) \odot x_2],$$
$$\frac{\partial \tilde{Z}}{\partial x_2} = (-6,3,3) \oplus \frac{1}{2}[(-4,4,12) \odot x_1 \oplus (6,14,6) \odot x_2]. \quad (24)$$

Step 3: Construct the problem

$$\min \tilde{Z} = (-3.5,1,5)_{LR} \odot x_1 \oplus (-4,9,9)_{LR} \odot x_2 \quad (25)$$

Subject to:

$$(1,1,1)_{LR} \odot x_1 \oplus (1,2,2)_{LR} \odot x_2 \preccurlyeq (3,5,1)_{LR},$$
$$(-2,2,6)_{LR} \odot x_1 \oplus (2,1,1)_{LR} \odot x_2 \preccurlyeq (3,5,1)_{LR}, \quad (26)$$
$$(2,2,2)_{LR} \odot x_1 \oplus (1,0,0)_{LR} \odot x_2 \preccurlyeq (3,2,2)_{LR}, x_1, x_2 \geq 0.$$

Then, problem (25) can be rewritten as:

$$\min \tilde{Z} = (-4.5,-3.5,1.5) \odot x_1 \oplus (-13,-4,5) \odot x_2 \quad (27)$$

Subject to:

$$(0,1,2) \odot x_1 \oplus (-1,1,3) \odot x_2 \preccurlyeq (-2,3,4),$$
$$(-4,-2,4) \odot x_1 \oplus (1,2,3) \odot x_2 \preccurlyeq (-2,3,4), \quad (28)$$

$(0,2,4)\odot x_1 \oplus (1,1,1)\odot x_2 \preccurlyeq (1,3,5), x_1, x_2 \geq 0.$

Using the ranking method defined in definition 3, the problem (27) becomes:

$$\min \hat{Z} = -2x_1 - 3.9975x_2 \tag{29}$$

Subject to:

$$x_1 + x_2 \leq 2;$$
$$-x_1 + 2x_2 \leq 2; \tag{30}$$
$$2x_1 + x_2 \leq 3, x_1,\ x_2 \geq 0.$$

The optimal solution of problem (21) is $\overline{x} = \left(\frac{2}{3}, \frac{4}{3}\right)$.

Step 4: Expand the objective function of problem (17) to the Taylor series at the solution$\overline{x} = \left(\frac{2}{3}, \frac{4}{3}\right)$, to obtain the fully fuzzy linear programming and then linear programming as:

$$\min \hat{Z} = -2.5x_1 - 2.75x_2 \tag{31}$$

Subject to:

$$x_1 + x_2 \leq 2;$$
$$-x_1 + 2x_2 \leq 2; \tag{32}$$
$$2x_1 + x_2 \leq 3, x_1,\ x_2 \geq 0.$$

The optimal solution of problem (21) is $\underline{x} = \left(\frac{2}{3}, \frac{4}{3}\right)$,

Step 5: The two solutions$\overline{x}$, and $\underline{x}$ are consistent, the optimal solution is $x^* = \left(\frac{2}{3}, \frac{4}{3}\right)$, and the fuzzy optimum value is $\tilde{Z} = (-6.444, 9.556, 12.889)_{LR}$.

5. CONCLUSION

In this paper, a new approach is proposed on solving fully fuzzy quadratic programming in which all the cost coefficients, constraints coefficients, and right hand side are characterized by $L - R$ fuzzy numbers. The approach can easily be applied to solve any QP problem. Through this approach, the FFQP problem is converted into the fully fuzzy linear programming and hence into linear programming with an arbitrary initial point. The advantages of the approach are differs from the others methods in computational step, easier than the other method that can be solved algebraically, the final solution can be obtained rapidly, and implemented in various types of nonlinear programming.

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