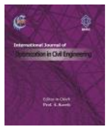
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2020; 10(4):553-570



A RELIABILITY APPROACH TO COMPARE OPTIMAL SEISMIC DESIGNS OF MULTIPLE-TUNED-MASS-DAMPER

Y. Naserifar and M. Shahrouzi*, †
Civil Engineering Department, Faculty of Engineering, Kharazmi University, Tehran, Iran

ABSTRACT

Passive systems are preferred tools for seismic control of buildings challenged by probabilistic nature of the input excitation. However, other types of uncertainty still exist in parameters of the control device even when optimally tuned. The present work concerns optimal design of multiple-tuned-mass-damper embedded on a shear building by a number of meta-heuristics. They include well-known genetic algorithm and particle swarm optimization as well as more recent gray wolf optimizer and its hybrid method embedding swarm intelligence. The study is two-fold: first, optimal designs by different meta-heuristics are compared concerning their reduction in structural seismic responses; second, the effect of uncertainty in *Multi-Tuned-Mass-Damper* parameters, is studied offering new reliability-based curves. *Monte Carlo Simulation* is employed to evaluate failure probabilities. A variety of structural responses are assessed against seismic excitation including maximal displacement, velocity and acceleration. It is declared that the best algorithm for efficiency and effectiveness has not coincided the best based on the reliability traces. Such traces also show that in a specific range of limit-states, algorithm selection has a serious effect on the reliability results. It was found even more than 35% and depends on the response type.

Keywords: seismic control; tuned mass damper; hybrid meta-heuristic algorithm; swarm intelligence; grey wolf optimizer; Monte Carlo simulation.

Received:10 July 2020; Accepted: 3 October 2020

1. INTRODUCTION

Reduction of structural responses due to seismic excitation can be favored by passive control of buildings. Such a control strategy relies on the embedded devices to the structural system

*Corresponding author: Civil Engineering Department, Faculty of Engineering, Kharazmi University, Tehran and Karaj, Iran
†E-mail address: shahruzi@khu.ac.ir (M. Shahrouzi)

that perform their tasks without the need for any external commands or energy sources during the earthquake. *Tuned Mass Damper,* TMD denotes one of the most popular devices in the category of passive seismic control; studied by several researchers [1–4].

Multiple Tuned Mass Damper, MTMD has already been introduced by investigators to efficiently face distributed-frequency excitations in the broad-band domain exhibiting superior performance over single TMD with similar mass. Single dampers may amplify higher modes on the consequence of the interference effect where coupling exists between the main mode and higher modes. Den Hartog has revealed pioneering research on optimizing TMD for an undamped equivalent *Single Degree Of Freedom* (SDOF) structure [5]. Jangid offered a methodology to find efficient parameters of MTMDs in an undamped structure subjected to harmonic excitation [6]. The method minimized displacements of the main system by means of a numerical technique. Wu and Chen [7] proposed a design procedure classifying MTMD into several groups, each of them covering a number of distributed dampers at different distinct levels. Hoang and Warnitchai [8] proposed a method for design of MTMD to minimize vibration response of linear *Multi-Degrees of Freedom* (MDOF) structures applying a numerical procedure.

Optimization of TMD parameters has been an active research field in recent decades [4,9–12]. In this regard, non-gradient based optimization methods have received more interest over conventional MP algorithms [13–15]. Meta-heuristics constitute a vast subset of zero-order methods; usually inspired by natural phenomena [16]. Some of the most popular ones are Genetic Algorithm [17], Simulated Annealing [18], Particle Swarm Optimization [19], Harmony Search [20], Charged System Search [21], Artificial Immune Systems [22], Colliding Bodies Optimization [23] and Grey Wolf Optimizer [24]. Hybrid algorithms are also offered for better performance in specific problems [25–29].

Another important issue in seismic control by TMD, is the effect of uncertainties on the desired performance. It is concerned here-in-after; however, several works have applied optimization regardless of this issue [6–9,29]. Since the structural model is more complicated than can be practically assessed by symbolic relationships; sampling methods are suited for this purpose [30]. In the present work, *Monte Carlo Simulation*, MCS is employed as it is well-studied and trusted in literature. In addition, a novel series of reliability curves are offered to compare optimal designs of TMD by different optimization methods including *Genetic Algorithm* and *Particle Swarm Optimization* as famous representatives of evolutionary computing and swarm intelligence; in addition to *Grey Wolf Optimizer* as a more recent algorithm and its hybrid variant with *Particle Swarm*; called PSOGWO. The results are derived on a literature benchmark structure.

2. GOVERNING EQUATION OF MOTION

Consider a $q-story$ building model under base excitation of $\ddot{y}_g$. Each damper of MTMD with its stiffness and damping is modeled in parallel on the last story of the structure.

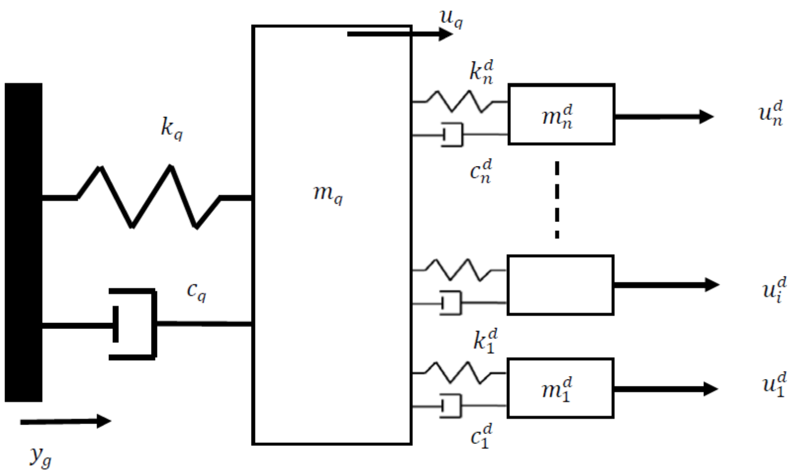


Figure 1. Schematic MTMD embedded on a model, depicted for sample case of $q = 1$

Assuming just one horizontal degree of freedom for every damper in MTMD, the entire model will have $q + n$ degrees of freedom. Consequently, the equation of motion is given in the matrix form by:

$$M\ddot{U}_{(t)} + C\dot{U}_{(t)} + KU_{(t)} = \ddot{y}_{g(t)} M\, e, \quad e_{(l+n)\times 1} = \left[-1 -1 -1 \ldots -1\right]^T \tag{1}$$

The scalar base-excitation on the effective force vector is applied using the influence vector;e multiplied by the mass matrix; M. The corresponding total stiffness and damping matrices are denoted by K and C, respectively. The stiffness matrix has the following pattern:

$$[K] = \begin{bmatrix} k_1 & -k_1 & & \ldots & & & & & & \\ & k_1 + k_2 & -k_2 & & & & & & & \\ & & k_2 + k_3 & \ldots & -k_{q-1} & & & & & \\ & & & & k_{q-1} + k_q & -k_q & & & & \\ & & & & & k_q + \sum_{i=1}^{n} k_i^d & -k_1^d & -k_2^d & \ldots & -k_n^d \\ & Sym. & & & & & k_2^d & & & \\ & & & & & & & k_3^d & & \\ & & & & & & & & \ldots & \\ & & & & & & & & & k_n^d \end{bmatrix} \tag{2}$$

The damping matrix has the same pattern as the stiffness matrix; while the mass matrix is diagonal. Such an equation of motion is solved under dynamic time-history excitation by the Wilson-theta's numerical analysis method.

3. OPTIMIZATION PROBLEM

The MTMD optimization under earthquake excitation is stated using a function of essential parameters in the equation of motion as follows.

Find:

$$\mathrm{X} = \left\{ k_1^d, c_1^d, \ldots, k_n^d, c_n^d \right\}^{\mathrm{T}} \tag{3}$$

to minimize:

$$J = \max_t \left\{ \left| u(t) \right| \right\} \tag{4}$$

subject to:

$$\begin{aligned} & 0 \le k_i^d \le k_{\max}^d \qquad i = 1, 2, \ldots, n \\ & 0 \le c_i^d \le c_{\max}^d \qquad i = 1, 2, \ldots, n \end{aligned} \tag{5}$$

The cost function J denotes maximum displacement of the structural model with respect to the ground within the duration of oscillation: $0 \sim t_{max}$. n stands for the number of TMD's embedded at the roof level. The damping and the stiffness of the i^{th} TMD are denoted by c_i^d and k_i^d, respectively.

4. UTILIZED ALGORITHMS

4.1 Particle swarm optimization

Particle Swarm Optimization, PSO, is a pioneering meta-heuristic in the category of directional search [31]; applied to several engineering problems since introduced by Kennedy and Eberhart [19]. It is simulated by movements of some artificial birds or particles in a D-dimensional search space. Suppose $X_i = (x_{i1}, x_{i2} \ldots, x_{iD})$ and $X_{Pbest,i} = (p_{i1}, p_{i2} \ldots, p_{iD})$ denote the current position of the bird and its best over experienced locations of it up to the current iteration, l, respectively. The latter simulates cognitive behavior of birds while the social term utilizes position of the best bird in the entire swarm; denoted by $X_{Gbest,i}$. Employing the time step of unity for computer simulation, the velocity term will represent changing in the position of each bird by $V_i = X_{i,new} - X_i$. Accordingly, PSO is presented via the following relations.

$$V_i^{l+1} = wV_i^l + c_1 r_1 \left(X_{Pbest,i}^l - X_i \right) + c_2 r_2 \left(X_{Gbest,i}^l - X_i \right) \tag{6}$$

$$X_i^{l+1} = X_i^l + V_i^{l+1} \tag{7}$$

The first equation emanated from the velocity vector includes three parts. The first part indicates inertia term regarding the previous velocity direction, the second or cognitive part, implies the personal experience of the particle and the third is the social term representing the cooperation among all particles. According to such PSO relations of particle movements, c_1, c_2 and w are the corresponding control parameters in addition to the population size and number of iterations (or function calls). r_1 and r_2 are random numbers uniformly generated between zero and one.

4.2 Genetic algorithm

Since formally introduced by Holland [17], several variants of the *Genetic Algorithm* are extensively used in finding optimal solutions for computational problems. In terms of their similarity to the biological reproduction processes, genetic algorithms are recognized as a major subset of evolutionary computation. Inherent randomness in many genetic operators is akin to evolution.

Any genetic algorithm is normally initiated with a random initial population deemed as including possible solutions to the problem. The method works with encoded/genotypic space that enables special explorative behavior [32]. It employs mutation and crossover thresholds as control parameters to effectively balance between the exploration and exploitation. GA operators include fitness-based selection, crossover and mutation in every new generation until the terminating iteration is reached. The method of encoding candidate solutions into chromosomes or what the fitness function is actually measuring, affects the ultimate efficiency of the genetic algorithm, to a large extent [33].

4.3 Grey wolf optimizer

Simulating behavior of grey wolves in catching their prey, *Grey Wolf Optimizer*, GWO, is introduced [24]; in which the search agents are distinguished via different kinds. Grey wolves' dominant hierarchy style-of-life is such that each pack of wolves is led by a leader entitled *Alpha*. The alpha's direct subordinate, called *Beta* is the senior counselor of alpha in making critical decisions. Beta is also responsible for successful publication of alpha's commands throughout the pack and providing alpha some helpful feedback. The lowest order among grey wolves belongs to *Omegas* which are the last to eat the prey. The remaining wolves are called *Delta* who are responsible for hunting, caretaking, scouting and so forth. Such a hierarchy of dominance is depicted in Fig. 2.

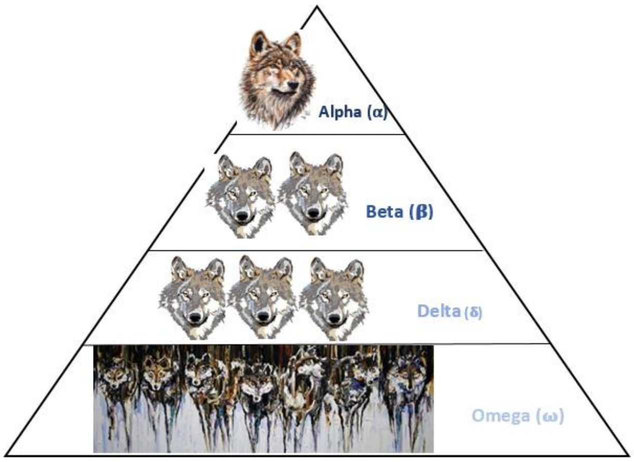


Figure 2. Social hierarchy of the grey wolves

GWO is a population-based algorithm to iterate for l^* times. It searches the design space via three stages: encircling, hunting, attacking and searching for the prey. The algorithm assorts fitness of solutions based on the concept of grey wolves' pack hierarchy, i.e. the fittest is considered the alpha while the next two best solutions are beta and delta, respectively. Omega is the name given to the remaining solutions. The method simulates the encircling stage during the hunting process by the wolves via the following relation; given for every j^{th} component [24]:

$$X_j^{(l+1)} = X_j^{(l)} - \left| C_j X_{p,j}^{(l)} - X_j(t) \right| A_j \tag{8}$$

At every iteration l, $X_{(l+1)}$ and $X_{(l)}$ denote the next candidate position of the wolf and its current position, respectively. The vector X_{p} stands for the prey position. A and C are auxiliary vectors in the aforementioned relations; given by:

$$A_j = (2\,rand - 1)\,a \tag{9}$$

$$C_j = 2\,rand \tag{10}$$

where $rand$ stands for a random generator function in range 0 to 1. The parameter a plays the role of a decreasing factor as l increases to its prescribed maximum; l^*.

$$a = 2(1 - \frac{l}{l^*}) \tag{11}$$

Based on the position of the pray (X^*, Y^*), the grey wolf's position (X, Y) is updated. The updated position of the best grey wolf can be achieved through the adjustment of the vectors A and C. Fig. 3 shows $2D$ position vectors and the next possible positions of the grey wolf.

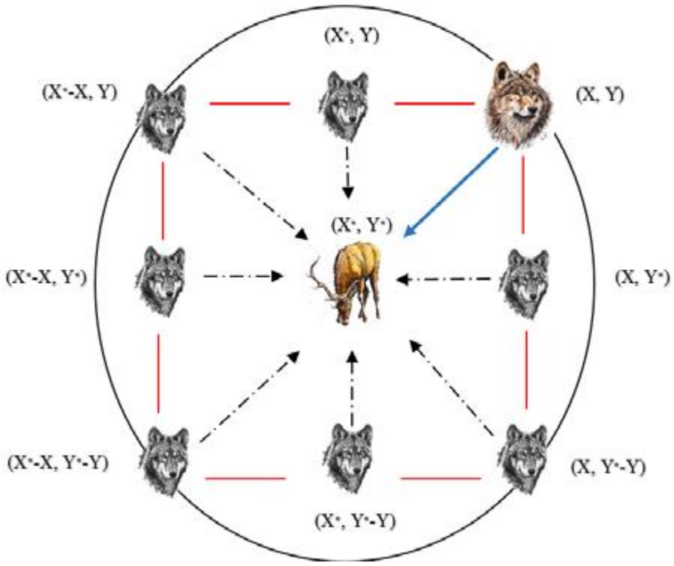


Figure 3. Wolves position vectors and alternatives in two-dimensions

In the hunting stage, information about position of the prey is obtained by cooperative work of the alpha, beta, and delta wolves. Given the fact that these wolves are representatives of the best three solutions of the problem, omega (the remaining) wolves' will be positioned, subsequently. Such new positions of the alpha, beta, and delta about the prey are given by Y_1, Y_2 and, Y_3.

$$\begin{aligned} Y_j^1 &= X_j^{(l)} - \left| C_{1,j}\, X_{\alpha,j}{}^{(l)} - X_j^{(l)} \right| A_{1,j} \\ Y_j^2 &= X_j^{(l)} - \left| C_{2,j}\, X_{\beta,j}{}^{(l)} - X_j^{(l)} \right| A_{2,j} \\ Y_j^3 &= X_j^{(l)} - \left| C_{3,j}\, X_{\delta,j}{}^{(l)} - X_j^{(l)} \right| A_{3,j} \end{aligned} \tag{12}$$

$$X_j{}^{(l+1)} = \frac{1}{3}(Y_j^1 + Y_j^2 + Y_j^3) \tag{13}$$

Equation 13, shows estimated position of the prey so that omega wolves can be directed around it.

In attacking the prey, i.e. when $|A| < 1$ each wolf updates its position between its current position and the position of the prey. At the time of searching for the prey, the alpha, beta, and delta may follow divergent directions but they come together when attacking the prey.

It is also needed to prevent getting trap in local optima by letting the wolves move away from the prey; that occurs in the case of $|A| > 1$.

4.4 Hybridization of particle swarm optimization and grey wolf optimizer

Inspired originally by the simulation of the social behavior of animals searching for food and hunting, Particle Swarm Optimization with exploitation and Grey Wolf Optimizer are hybridized and merged using a low-level co-evolutionary mixed hybrid to form the Hybrid *Particle Swarm Optimization* and *Grey Wolf Optimizer* (PSOGWO). The modified set of governing equations over the search space in order to simulate simultaneous PSO and GWO methods are introduced and updated as follows regarding the position vector and space updating velocity variants [34].

$$\begin{aligned} Z_j^1 &= X_j^{(l)} - \left| C_{1,j} X_{\alpha,j}^{(l)} - X_j^{(l)} \right| A_{1,j}.w \\ Z_j^2 &= X_j^{(l)} - \left| C_{2,j} X_{\beta,j}^{(l)} - X_j^{(l)} \right| A_{2,j}.w \\ Z_j^3 &= X_j^{(l)} - \left| C_{3,j} X_{\delta,j}^{(l)} - X_j^{(l)} \right| A_{3,j}.w \end{aligned} \tag{14}$$

$$V_i^{l+1} = w\{V_i^l + c_1 r_1 \left(Z^1 - X_i^l\right) + c_2 r_2 \left(Z^2 - X_i^l\right) + c_3 r_3 \left(Z^3 - X_i^l\right)\} \tag{15}$$

$$X_i^{l+1} = X_i^l + V_i^{l+1} \tag{16}$$

5. RELIABILITY ASSESMENT

Several uncertainties exist in structural parameters including construction materials, external loads, geometry, etc. Due to structural reliability theory, it is now possible to formulate such uncertainties and consider them in the design. The reliability analysis methods can be distinguished in some major categories; one assumes symbolic relations for marginal functions while the other is most suitable for non-symbolic evaluations of failure probabilities.

First Order / *Second Order Reliability Methods* (FORM/SORM) fall in the first category supported by considerable theoretical relations. A high-order function may be evaluated by series expansion to be suited for either FORM or SORM; even when applicable in some simple models they may bring about a degree of approximation.

The second category; however, relies on sampling of design points to declare limit-state function. The most famous procedure in this class is *Monte Carlo Simulation.* MCS can be easily programmed for implicit function evaluation and behaves like simulation of an experiment with random numbers. First, random numbers are generated with desirable (e.g. normal) distribution to simulate uncertainty in the parameters of the problem. Then, every such sample constitutes a design point that can be further evaluated by structural analysis to see whether it passes the limits or not. Probability density function is given here as [35–37]:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right),(\mu,x)\in R,\sigma>0 \tag{17}$$

where *x*, μ and are random variables, mean and standard deviation, respectively. In order to calculate the failure probability of a system by MCS, the failure limit should be declared a priori. Then, MCS is performed for each sample to declare whether it has failed or not. Dividing the number of failed samples; N_f by the total amount of samples; N the probability of failure; P_f is resulted by:

$$P_f=\frac{N_f}{N} \tag{18}$$

The reliability value for the corresponding failure probability is defined here as:

$$P_r=1-P_f \tag{19}$$

6. NUMERICAL RESULTS

The ten-story shear building model of Fig. 4 is considered as a case study; already addressed in literature [38]. Mass, damping and stiffness of each story are taken 360000*kg*, 6.2*MNs/m* and 650*MN/m*, respectively. The structure is subjected to the accelerogram of Fig. 5. A program has been developed to analyze the shear frame with a control system employing the Wilson-Theta's numerical procedure. It is verified by the results presented in literature [38] as reported in Table 1. The task has been accomplished on a platform with Core 2 Duo CPU; 2.66 GHz. Consequently, critical story responses are evaluated for the controlled and uncontrolled conditions.

The control system considered for this structure is an MTMD consisting of 10 TMD's embedded in parallel distribution on the top level of the building. Stiffness and damping parameters of each TMD are unknowns; then for 10 TMD's, there are 20 design variables for this optimization problem.

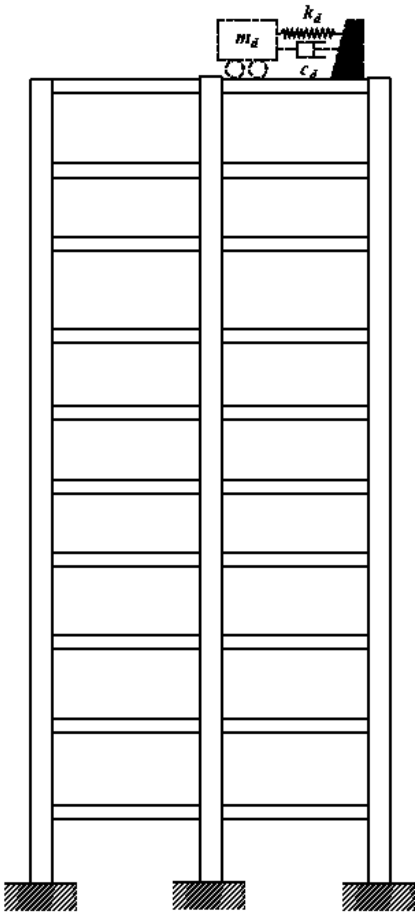


Figure 4. 10-Story shear building with MTMD at the roof level

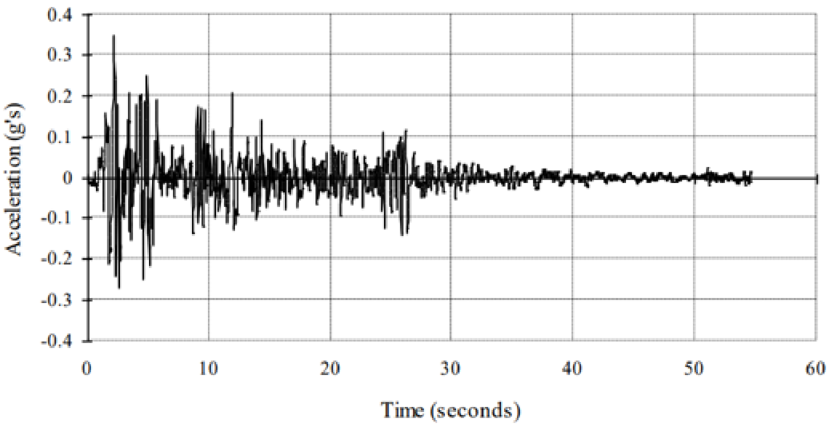


Figure 5. Time-history record for El-Centro 1940 Earthquake

Table 1: Validation of the provided analysis program with literature results

Story	**Displacements (cm)**			
	With TMD		**Without TMD**	
	Hadi &Arfiadi 1998 [38]	**Present study**	**Hadi &Arfiadi 1998 [38]**	**Present study**
1	1.90	1.87	3.10	3.05
2	3.70	3.65	6.00	5.96
3	5.80	5.30	8.70	8.67
4	6.80	6.79	11.20	11.13
5	8.20	8.12	13.30	13.29
6	9.40	9.34	15.10	15.11
7	10.40	10.39	16.60	16.58
8	11.30	11.26	17.70	17.68
9	11.90	11.87	18.40	18.42
10	12.20	12.19	18.80	18.79

In order to find the characteristics of the control system, GA, PSO, GWO and PSOGWO are considered as solution algorithms. Table 2 reveals the applied control parameters for each algorithm. The objective is to minimize lateral displacements of the frame under prescribed seismic excitation. Mass ratio of the control system is fixed to 3% of the building mass. Bounds on K_d and C_d values, are given in Table 3. All the algorithms are run up to 500 iterations; and the best achieved objective values are demonstrated in Fig. 6. It is observed that the highest convergence rate belongs to GWO while PSO has obtained the best final cost as the objective function. PSOGWO has revealed similar convergence to GWO but GA has shown the lowest convergence rate. Further statistical results are reported in Table 4.

Table 2: Applied control parameters

Algorithm	**Number of Iterations**	**Population size**	**Inertial Weight**	**Cognitive Parameter C_1**	**Social Parameter C_2**	**Crossover Rate**	**Mutation Rate**
GA	500	25	-	-	-	1.0	0.5
PSO	500	25	1.0	1.5	2.0	-	-
GWO	500	30	-	-	-	-	-
PSOGWO	500	30	1.0	0.5	0.5	-	-

Table 3: Upper bounds on stiffness and damping of TMD's

TMD property	**limit**
k_{max}^{d}	$4000\ (\frac{kN}{m})$
c_{max}^{d}	$1000\ (\frac{kN.s}{m})$

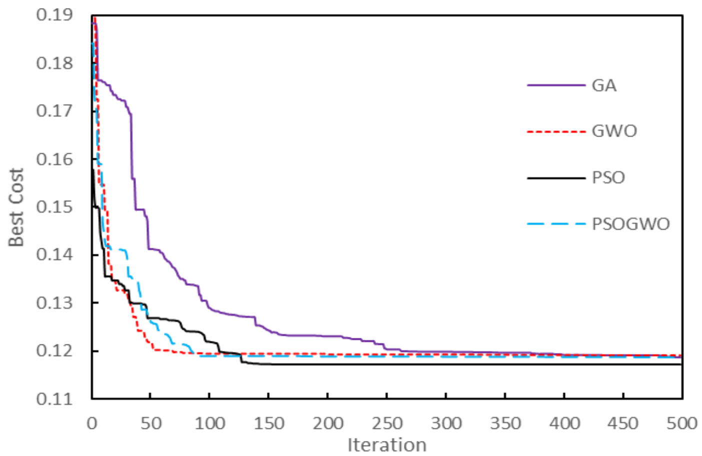


Figure 6. Convergence curves of GA, GWO, PSO and PSOGWO for MTMD design

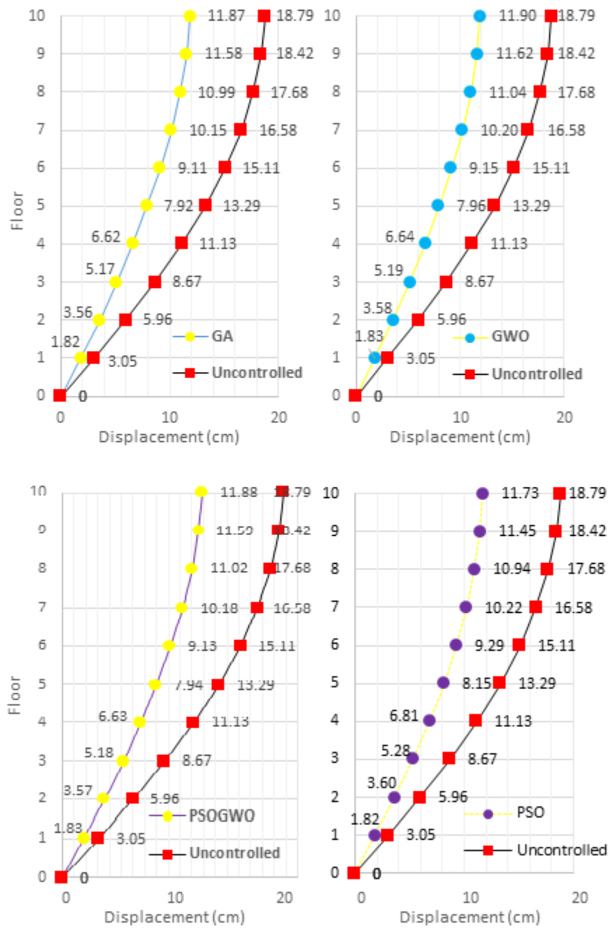


Figure 7. Absolute uncontrolled and controlled displacements by different algorithms

Table 4: Statistical results of the treated algorithms for MTMD design

Method	Best cost	Mean cost	COV
GA	0.118700	0.127390	0.014898
PSO	0.117262	0.120091	0.006290
GWO	0.118986	0.121265	0.008600
PSOGWO	0.118754	0.121592	0.008398

* COV: Coefficient Of Variation

GA, GWO, and PSOGWO optimization algorithms have different convergence rates but not so different final costs. The employed PSO has exhibited better mean and best results with proper COV compared to the others.

Comparison of resulted maximum story sways in Fig. 7, reveals considerable reduction in the structural response for the optimally controlled model with respect to the uncontrolled structure. The greatest value of such a reduction has occurred at the roof level; meanwhile, it is realized that in this example PSO has been superior to the others.

The next issue to investigate is how uncertainties may affect such optimal results. Reliability analysis is inevitable to address this question. The structural reliability is derived for every designed control system by Monte Carlo Simulation (MCS); in which random parameters are generated due to a prescribed probability distribution.

In this regard, stiffness and damping of MTMD in the employed control system are assumed to be random variables obeying continuous normal distribution. For every uncertain parameter 5000 samples are generated; each one being a new model of structural control system with the corresponding seismic displacement, acceleration and velocity responses.

Moreover, in order to calculate P_r, several structural models are sampled and checked for passing the limit-state against the aforementioned criteria. Then, reliability of the structure in every its specific response is evaluated by Eq. (19). According to Table 5, the range of ultimate limit-states for all the structural responses is taken a value between the maximum uncontrolled structural displacement D, Velocity V and acceleration A as the upper bounds and fifty percent of them as the corresponding lower bounds. The uncontrolled responses are obtained in this case study as $D = 0.1879\ m$, $V = 0.0149\ m/s$ and $A = 0.0973\ m/s^2$.

Table 5: Applied limit-state bounds on the structural responses

Response type	Maximum response of the uncontrolled Structure	Ultimate limit-State bound	
		Lower bound	Upper bound
Displacement	D	0.5 D	D
Velocity	V	0.5 V	V
Acceleration	A	0.5 A	A

To assess reliability of optimal designs with respect to critical displacement, velocity or acceleration responses, a novel set of reliability traces, is offered. Any point in the curve of each specific structural response is generated by calculating the reliability due to MCS for a certain percentage of maximum response within its range. The curve is completed when all its points are generated by several MCS operations for the optimal design by each algorithm. The parameter on the horizontal axis should be discretized to a finite number of points instead of treating it as a continuous variable with infinite (impractical) values. It is inevitable in the present structural problem, in the absence of a symbolic relation for the limit-state function; as it should be distinctly evaluated for every MCS sample (structural model).

Fig. 8 shows such a trace regarding displacement response in the range 0.5D to D. The reliability curves for optimal designs by all 4 treated algorithms are plotted vs. each other, for the sake of better comparison. Although GA's best design has not revealed the lowest cost, its reliability trace stands over the others (PSO, GWO and PSOGWO) in most of displacement points. For any such points, the reliability curve of PSOGWO has been lower than the others while those of PSO and GWO are closer to each other.

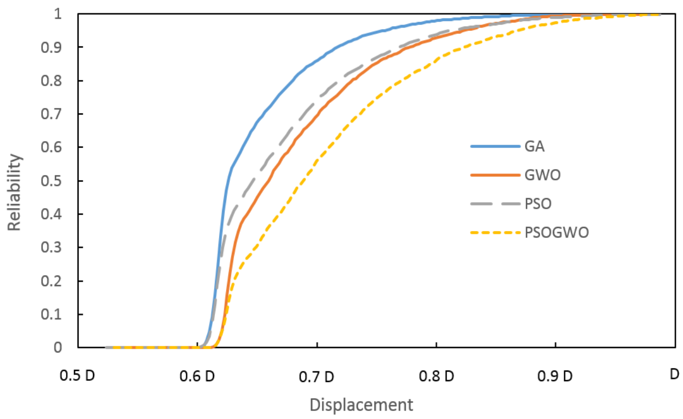


Figure 8. Reliability traces for different optimal designs vs. displacement response

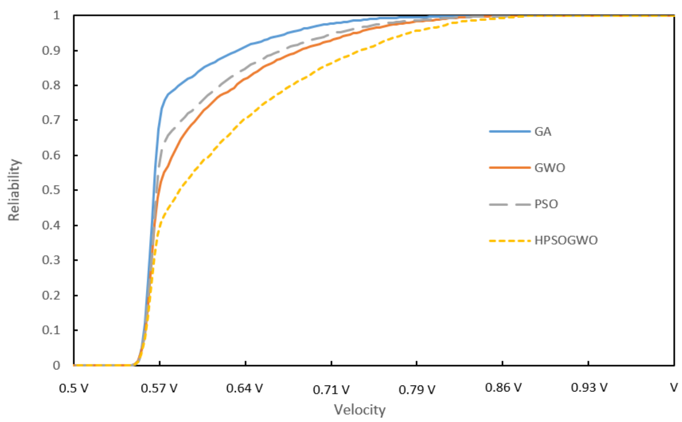


Figure 9. Reliability traces for different optimal designs vs. velocity response

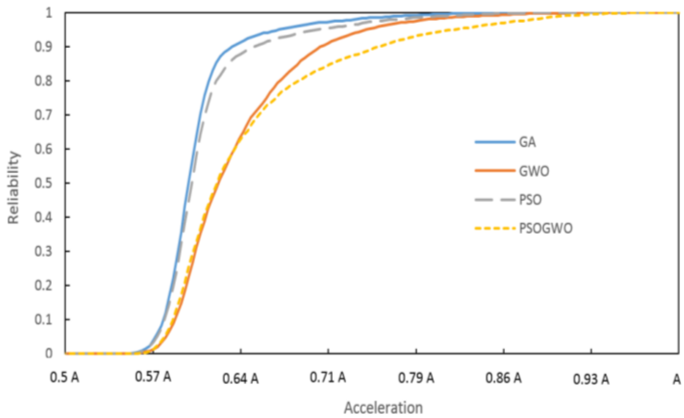


Figure 10. Reliability traces for different optimal designs vs. acceleration response

Similar trend is observed for reliability traces vs. the velocity response in Fig. 9. It is while for acceleration response in Fig. 10, GWO has revealed closer curve to PSOGWO than the others. Again, it is declared that the best algorithm regarding the optimality of final solution has not coincided with the best regarding reliability preference.

Table 6: Reliability variation vs. limit-state variation for the treated algorithms

	limit-State Range	**Reliability**				**Reliability variation**
		GA	PSO	GWO	PSOGWO	
Displacement	0.63D-0.81D	reliabilities at 0.63D				0.13-0.35
		0.55	0.42	0.20	0.20	
Velocity	0.57V-0.75V	reliabilities at 0.57V				0.11-0.36
		0.76	0.65	0.55	0.40	
Acceleration	0.58A-0.75A	reliabilities at 0.62A				0.04-0.26
		0.87	0.83	0.53	0.53	

Table 7: Maximal absolute roof displacement, velocity and acceleration for the uncontrolled and optimally controlled structure by GA

Case	**Displacement (cm)**	**Velocity (cm/s)**	**Acceleration (cm/s^2)**
Controlled	11.87	0.79	5.94
Uncontrolled	18.79	1.44	9.73
Reduction (%)	37	45	39

Furthermore, such traces show that in a specific range of limit-states, algorithm selection has a serious effect on the reliability results. For example, at the range of (0.63D-0.81D) in Fig. 8, GA illustrates the best result, with a maximum at 0.81D. However, outside of this range, all algorithms approximately have closer results. It can also be noticed that the maximum difference between algorithm reliabilities has occurred at about 0.63D. For this point, GA reliability is 0.13 higher than PSO, 0.35 more than GWO and PSOGWO. Results of such a test for velocity and acceleration have been reported in Table 6. In addition, it reveals that the designed optimized MTMD decreases the lateral displacement of the highest elevation of the building by 37% (Table 7). Such a reduction is obtained 45% and 39% for velocity and acceleration responses, respectively.

7. CONCLUSION

Optimal design of MTMD for passive control of building structure against dynamic base-excitation was treated by a number of meta-heuristics; i.e. GA as an evolutionary method, PSO that is a swarm algorithm and GWO being a population-based method with different kinds of search agents. PSOGWO was also employed as a hybrid combination of the last two. In this type of optimization that each of 10 TMD’s was tuned by the algorithm; it was observed that the employed PSO can reveal lower cost with respect to the others. GA has the lowest convergence rate while GWO and PSOGWO have stood on the middle ranks for the applied set of control parameters.

The optimal designs were further subjected to a reliability study. Consequently, for every such design, sampling by MCS was performed over the discretized response axis; once for displacement response and the other times for velocity and acceleration. In this regard, a novel set of curves was offered and utilized to compare the aforementioned designs from the reliability point of view.

According to the results of this study, it was found that not only the most effective algorithm may not reveal the most reliable design but also the first rank in the reliability can be associated with the less efficient optimization method. In our study, the first rank belongs to GA among the others; that exhibited a lower convergence rate but higher reliability value (lower failure probability calculated by MCS) over a vast range of the discretized limit-state parameter. Such a rank was stable for acceleration and velocity responses in addition to displacement. Hence, it cannot be recommended to suppress reliability study in the design of such a passive control system.

Due to specific results of this case study, maximum superiority of GA in P_r over the others were observed (0.13-0.35), (0.11-0.36) and (0.04-0.26) for the displacement limit-state in the range of (0.63D-0.81D), for the velocity within (0.57V-0.75V), and for the acceleration within (0.58A-0.75A), respectively. Outside such range, the reliability difference was lower between optimal designs by GA, PSO, GWO and PSOGWO. In another word, the condition that how much can rely on the effectiveness of optimal designs for reliability, depends on the range of the parameter in hand regarding the proposed reliability curves. Applying wider range of earthquake records to more structural models with a variety of optimization algorithms can, of-course, be a future scope of research.

REFERENCES

1. Chang JCH, Soong TT. Structural control using active tuned mass dampers, *J Eng Mech Div* 1980; **106**: 1091–8.
2. Soong TT, Reinhorn AM. An overview of active and hybrid structural control research

in the US, *Struct Des Tall Build* 1993; **2**: 193–209.
3. Pourzeynali S, Lavasani HH, Modarayi AH. Active control of high rise building structures using fuzzy logic and genetic algorithms, *Eng Struct* 2007; **29**: 346-57.
4. Mohebbi M, Joghataie A. Optimal TMDs for improving the seismic performance of historical buildings, *Sci Iran* 2016; **23**: 79–90.
5. Den Hartog JP. *Mechanical Vibrations*, Courier Corporation, 1985.
6. Jangid RS. Optimum multiple tuned mass dampers for base-excited undamped system, *Earthq Eng Struct Dyn* 1999; **28**: 1041-9.
7. Chen G, Wu J. Optimal placement of multiple tune mass dampers for seismic structures, *J Struct Eng* 2001; **127**: 1054-62.
8. Hoang N, Warnitchai P. Design of multiple tuned mass dampers by using a numerical optimizer, *Earthq Eng Struct Dyn* 2005; **34**: 125-44.
9. Li C. Optimum multiple tuned mass dampers for structures under the ground acceleration based on DDMF and ADMF, *Earthq Eng Struct Dyn* 2002; **31**: 897–919.
10. Leung AYT, Zhang H, Cheng CC, Lee YY. Particle swarm optimization of TMD by non-stationary base excitation during earthquake, *Earthq Eng Struct Dyn* 2008; **37**: 1223–46.
11. Shahrouzi M, Nouri G, Salehi N. Optimal seismic control of steel bridges by single and multiple tuned mass dampers using charged system search, *Int J Civil Eng* 2017; **15**: 309-18.
12. Etedali S, Akbari M, Seifi M. MOCS-based optimum design of TMD and FTMD for tall buildings under near-field earthquakes including SSI effects, *Soil Dyn Earthq Eng* 2019; **119**: 36–50.
13. Belegundu AD, Arora JS. A study of mathematical programmingmethods for structural optimization. Part II: Numerical results, I*nt J Numer Method Eng* 1985; **21**: 1601-23.
14. Yang X. Nature-Inspired Metaheuristic Algorithms, 2010.
15. Kaveh A. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*, Switzerland, Springer, 2017.
16. Parpinelli RS, Lopes HS. New inspirations in swarm intelligence: A survey, *Int J Bio-Inspired Comput* 2011; **3**: 1–16.
17. Holland JH. Adaptation in natural and artificial systems : an introductory analysis with applications to biology, control, and artificial intelligence, 1975.
18. Kirkpatrick S, Gelatt CD, Vecchi MP. Optimization by simulated annealing, *Sci* 1983; **220**: 671–80.
19. Kennedy J, Eberhart RC. Particle swarm optimization, *Proccedings of the IEEE International Conference Neural Networks*, 1995.
20. Geem ZW, Kim JH, Loganathan G V. A New Heuristic Optimization Algorithm: Harmony Search, *Simulat* 2001; **76**: 60–8.
21. Kaveh A, Talatahari S. A novel heuristic optimization method: Charged system search, *Acta Mech* 2010; **213**: 267–89.
22. Farmer JD, Packard NH, Perelson AS. The immune system, adaptation, and machine learning, *Phys D Nonlinear Phenom* 1986; **22**: 187-204.
23. Kaveh A, Mahdavi VR. *Colliding Bodies Optimization: Extensions and Applications*, Springer, 2015.

24. Mirjalili S, Mirjalili SM, Lewis A. Grey Wolf Optimizer, *Adv Eng Softw* 2014; **69**: 46-61.
25. Talbi EG. Metaheuristics. Hoboken, NJ, USA: John Wiley & Sons, Inc, 2009.
26. Roozbeh Nia A, Hemmati Far M, Niaki STA. A hybrid genetic and imperialist competitive algorithm for green vendor managed inventory of multi-item multi-constraint EOQ model under shortage, *Appl Soft Comput J* 2015; **30**: 353-64.
27. Kaveh A, Talatahari S. Engineering Optimization with Hybrid Particle Swarm and Ant Colony Optimization 2009; *Asian J Civil Eng* **10**: 611-28.
28. Kaveh A, Bakhshpoori T, Afshari E. An efficient hybrid Particle Swarm and Swallow Swarm Optimization algorithm, *Comput Struct* 2014; **143**: 40–59.
29. Arfiadi Y, Hadi MNS. Optimum placement and properties of tuned mass dampers using hybrid genetic algorithms, *Int J Optim Civ Eng* 2011; 1: 167–87.
30. Rackwitz R. Reliability analysis - A review and some perspectives, *Struct Safe* 2001; **23**(4): 365–95.
31. Shahrouzi M. Pseudo-random Directional Search: a new heuristic for optimization, *Int J Optim Civ Eng* 2011; **1**: 341–55.
32. Kaveh A, Shahrouzi M. Simultaneous topology and size optimization of structures by genetic algorithm using minimal length chromosome, *Eng Comput* 2006; **23**: 644-74.
33. Goldberg DE. Genetic Algorithms in Search, Optimization & Machine *Learning*, Addison-Wesely Longman, Boston, 1989.
34. Singh N, Singh SB. Hybrid algorithm of particle swarm optimization and grey wolf optimizer for improving convergence performance, *J Appl Math* 2017; 1-15.
35. Fishman G. Monte Carlo: concepts, algorithms, and applications. 2013.
36. Borkowf CB, Gentle JE. Random number generation and monte carlo methods, *Technometr* 2000; **42**: 431.
37. Tranter WH, Sam K, Theodore S, Rappaport S, Kosbar KL. Principles of Communication Systems Simulation with Wireless Applications, 2004.
38. Hadi MNS, Arfiadi Y. Optimum design of absorber for MDOF structures, *J Struct Eng* 1998; **124**: 1272–9.