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OPTIMUM DESIGN OF SINGLE-LAYER DOME STRUCTURES USING A HYBRID CHARGED SYSTEM SEARCH AND TEACHING-LEARNING-BASED OPTIMIZATION

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ABSTRACT

The optimal design of dome structures is a challenging task and therefore the computational performance of the currently available techniques needs improvement. This paper presents a combined algorithm, that is supported by the mixture of Charged System Search (CSS) and Teaching-Learning-based optimization (TLBO). Since the CSS algorithm features a strong exploration and may explore all unknown locations within the search space, it is an appropriate complement to enhance the optimization process by solving the weaknesses with using another optimization algorithm's strong points. To enhance the exploitation ability of this algorithm, by adding two parts of Teachers phase and Student phase of TLBO algorithm to CSS, a method is obtained that is more efficient and faster than standard versions of these algorithms. In this paper, standard optimization methods and new hybrid method are tested on three kinds of dome structures, and the results show that the new algorithm is more efficient in comparison to their standard versions.

Keywords: dome structures; charged system search; optimum design; teaching-learning-based optimization; structural optimization.

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1. INTRODUCTION

The optimization of structures is a challenging task for engineers and designers. Structural engineers have been persuaded to impose new challenges in the field of structural analysis and design. The appropriate use of various sections for elements of structures is the main goal of designers to make an economical and reliable design. The size, topology and shape

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efficiency of structural components are the foremost major issues for designers to scale back the entire cost of the building. Aim to attenuate the load of the project by fulfilling the planning constraints by new methods of optimization algorithms.

Numerous meta-heuristic algorithms have been developed for finding optimum design of structures, that have already been based on the concepts of certain influence of physics and biology. Based on the principles used, these equations are divided into various categories. There are several well-known algorithms that are created on the idea of certain influence of nature within the "biological ecology and reproduction" class, like the Genetic Algorithm (GA) [1-10]. On the other hand, Teaching–Learning-Based Optimization (TLBO) works on the impact of a teacher's influence on learners. Like other natural-inspired algorithms, TLBO is additionally a population-based approach and uses a population of solutions to push towards a worldwide solution. The population is understood to be a community of learners or a category of learners. The TLBO process is split into two parts: the primary section consists of the 'Teachers Phase' and therefore the second section consists of the 'Learners Phase.' 'Teacher Phase' means learning from an instructor and 'Learner Phase' means learning by interaction between learners [11-13]. Other meta-heuristic optimization technique which can be used for optimal design of large scale structures as domes are [14-22].

Here we have used Charged System Search (CSS) which relies on the principles of Coulomb and Gauss from electrical phenomena and Newtonian Mechanics' regulations of movement. equations from electrical physics and quantum mechanics [23-25]. The CSS algorithm determines an amount of solutions, each of which is referred to charged particle (CP) and expected to act as charged scope and each (CP) can impose an electrical influence on the other elements (CPs). Such influences will also change the direction of all other CPs per the Newton’s 2nd law and following that the latest positions of the CPs have been decided.

This present work introduces a hybrid algorithm for optimum design of dome structures. In this paper, a contemporary algorithm is employed that blends CSS with TLBO as a modified charged system search. The combination is performed in order to increase the efficiency of the CSS by maintaining positive characteristics of TLBO as referred to as 'Teachers Phase' and 'Learner Phase'.

2. FORMULATION OF THE DOME DESIGN OPTIMIZATION PROBLEM

2.1 Optimum configuration of domes

Discovering an optimal cross section in domes, is the key to style an optimum dome structure. By optimum design in rise of the crown part and therefore the rings beneath, it is possible to define loading parameters. An appropriate model to point out the target function can be:

$$\text{Find} \qquad X = \{A_i, h, N_r, \;\; i = 1 : ng\} \tag{1}$$

$$\text{To minimize} \qquad W(X) = \sum_{i=1}^{ng} \gamma_i \, . A_i . \sum_{j=1}^{ni} L_j \tag{2}$$

Where X is the design vector representing all structural parameters; A_i is the element cross section of the design table of the ith quantity that is chosen from steel pipe categories of LRFD-AISC [26]. N_r is the total number of rings; ng represents the total count of size categories; $W(X)$ is the weight of the design; ni is the total number of elements in group i;L_j is the length of element j; h is the rise of the dome and γ_i is the material mass density. A_i shows the effect of cross-section on the load of the dome, and therefore the choice of various values for N_r also can change the dimensions of all other variables. The specification of the LRFD [26] and also the restrictions of the drift are known to be constraints for these structures. Constraints are often defined as:

Displacement constraint

$$\delta_i \leq \delta_i^{max} \quad i = 1,2, \ldots, \text{nn} \tag{3}$$

Constraint on each Interaction Formula

$$\text{For } \frac{P_u}{\phi_c P_n} < 0.02 \qquad \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1 \tag{4}$$

$$\text{For } \frac{P_u}{\phi_c P_n} \geq 0.02 \qquad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1 \tag{5}$$

Shear constraint

$$V_u \leq \emptyset_v V_n \tag{6}$$

where the δ_i is the displacement of the node i ; δ_i^{max} is an allowed displacement of the ith node; nn is the total number of nodes; ϕ_c is the resistance factor; P_u is the necessary strength; P_n is the nominal axial force; M_{ux} is the required flexural strength in X direction and M_{uy} is the same factor for Y direction; ϕ_b and is the factor of decrease of flexural resistance. V_u is the factored service load shear; V_n is the nominal shear strength and ϕ_v represents the resistance factor for shear.

3. COMBINED METHODS

3.1 Charged search system

The Charged System Search (CSS) algorithm relies on the principles of Coulomb and Gauss on electrical phenomena and Newtonian Mechanics' regulations of movement. This algorithm is often referred to as a multi-agent strategy, during which each agent may be a Charged Particle (CP). Every CP would be considered to become a charged sphere with radius a providing a consistent quantity charge density and is equal to

$$q_i = \frac{fit(i) - fitworst}{fitbest - fitworst} i = 1,2, \ldots, N \tag{7}$$

Where *fitbest* and *fitworst* are ideal and the worst of all the particles; *fit(i)* symbolizes the fitness of the agent *i* and *N* denotes the total number of CPs. The CPs could impose electrical influences on the others. The type of influences is attractive, and its magnitude for the CP situated inside the sphere is equivalent to the gap distance between the CPs, and for the CP situated beyond the sphere, it is inversely adequate to the square of the gap distance between the particles.

$$Fj = q_i \sum_{i,i \neq j} (\frac{q_i}{a^3} r_{ij}.i_1 + \frac{q_i}{{r_{ij}}^2}.i_2) P_{ij}(X_i - X_j) \quad \begin{cases} j = 1,2,\ldots,N \\ i_1=1 \;\; i_2=0 \Leftrightarrow r_{ij}<a \\ i_1=0 \;\; i_2=1 \Leftrightarrow r_{ij} \gg a \end{cases} \tag{8}$$

in which F_j is the resulting reaction force on the CP; r_{ij} is the separation distance between the two charged particles described as follows:

$$r_{ij} = \frac{||X_i - X_j||}{\left\|\frac{(X_i - X_j)}{2} - X_{best}\right\| + \varepsilon} \tag{9}$$

in which X_i and X_j are the positions of the i th and the j th CPs, respectively; from which X_{best} is the position of the best current CP, and where ε is a small positive value. The prior positions of the CPs are randomly determined within the search space and therefore the initial velocity of the charged particles is assumed to be zero. Here, p_{ij} defines the probability of every CP starting to move towards the opposite CPs as

$$\begin{cases} 1 \;\; \frac{fit(i) - fitbest}{fit(j) - fit(i)} > rand \; V \; fit\ (j) > fit(i) \\ \;\; 0 \qquad\qquad\qquad\qquad\qquad otherwise \end{cases} \tag{10}$$

The resulting forces and therefore the laws of motion decide the new position of the CPs. Now, each CP continues to move towards its new position as a result of the action of resultant forces and its previous velocity.

$$X_{j,new} = rand_{j1}.k_a.\frac{F_j}{m_j}.\Delta t^2 + rand_{j2}.k_v.V_{j,old}.\Delta t + X_{j,old} \tag{11}$$

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t} \tag{12}$$

Where $k_a$is the ratio of acceleration; k_v is the ratio of velocity for regulating the effect of the previous velocity; While $rand_{j1}$and $rand_{j2}$ are two random numbers, and 1 determines the rate of selecting a value in the new vector from the historic values stored in the CM, and

(1-CMCR) sets the rate of randomly selecting one value from the potential value range. The pitch adjustment operation is followed only after the value of the CM is selected. The value (1-PAR) sets the pace at which nothing is achieved. Here, "w.p." means "with probability".' For further details, the reader may refer to Kaveh and Talatahari [24, 25].

To have a discreet result, a rounding function is used which modify the magnitude of the outcome to the closest available discrete value as follows

$$X_{j,new} = Round(rand_{j1}.k_a.\frac{F_j}{m_j}.\Delta t^2 + rand_{j2}.k_v.V_{j,old}.\Delta t + X_{j,old}) \quad (13)$$

3.2 Teaching–learning-based optimization (TLBO)

Teaching – Learning-Based Optimization (TLBO) is a nature-based algorithm that has its own ability to properly tackle various optimization problems and it is proposed to supply an answer for continuous nonlinear functions with less calculation time and practical method. The TLBO procedure is attributed to the influence of an instructor's impact on the output of pupils within the class. During this case, performance is analyzed in terms of exam grades. The efficiency of the teacher affects the outcomes of the pupils. It is evident that knowledgeable teacher train learner in such how that they will have better performance in terms of their marks or test scores.

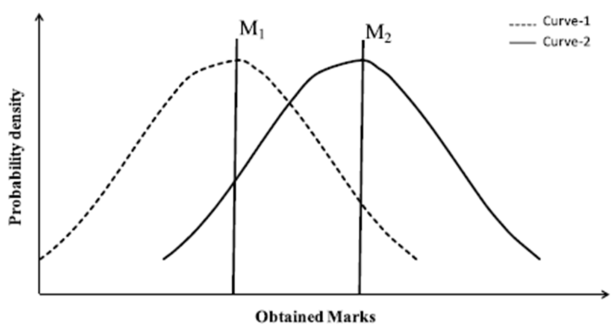


Figure 1. Distribution of marks received from pupils taught by two separate teachers

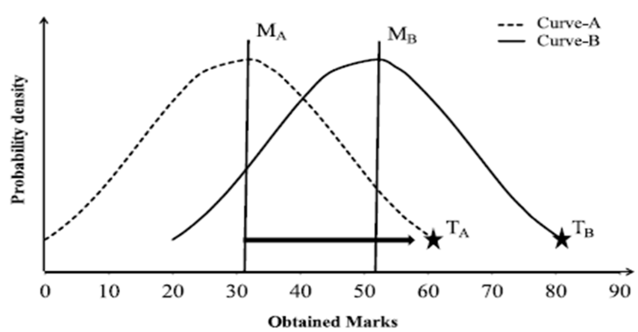


Figure 2. Model for the distribution of the marks obtained by a group of scholars

The standard pattern of the gathered marks was assumed, however in practical terms it can have skewness. Fig. 1 indicates the range of the marks received by the pupils in two separate classes assessed by the professors. Curves 1 and a couple of are the marks acquired by the scholars trained by the teachers T1 and T2, respectively. The traditional distribution research is described as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{14}$$

in which σ^2 is the variance, μ is the mean, and x is the amount for which the normal distribution function is required.

The key difference between the two tests is their mean (M_2 for Curve-2 and M_1 for Curve-1). A qualified teacher generates a higher mean for test scores. It can be seen from Fig. 2 that curve-3 is better than curve-1 and so it can be said that teacher T_2 is better than teacher T_1 in teaching. The teacher seeks to share his or her knowledge among the pupils, which successively would improve the extent of data of the whole class and permit the scholars to develop high marks or scores. Fig. 2, which indicates the graph for the marks scored for the pupils in the curve class-A getting a mean M_A. The instructor is perceived to be the most knowledgeable member of the society, such that the best learner is imitated as an instructor, as shown by T_A in Fig. 1. Teacher T_A tries to improve the level of students from M_A to M_B, at which point students require a new instructor of top quality than themselves. in this case the new teacher is T_B. There will then be a new curve-B with the new T_B instructor.

Fig. 3 showed that the TLBO algorithm divided into two sections. The first part consists of the 'Teacher Stage' and the second section consists of the 'Pupil Stage'.

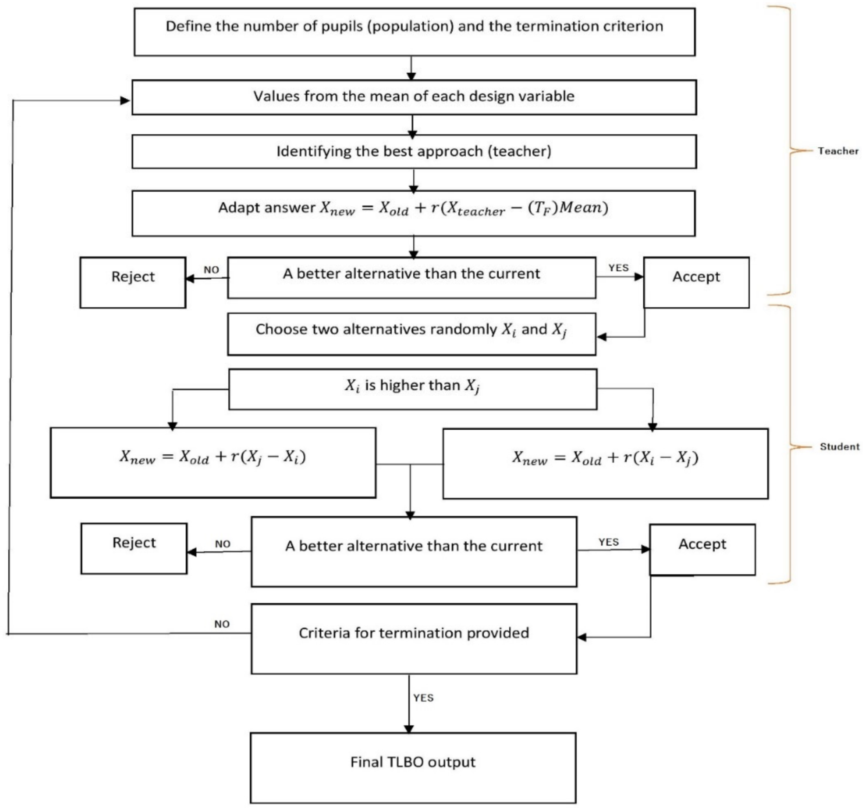


Figure 3. Flowchart of the TLBO

Teacher's stage

A successful teacher is the one who carries his/her learners up to his/her level of data. Yet it is impossible actually, so an instructor can only shift the category average to a particular degree counting on the category capacity. T_i is going to try to move M_i to his own stage of development, meanwhile the new mean is going to be T_i appointed as M_{new}. Like the one shown in Fig. 1, the mean of a class increases from M_A to M_B, usually depends on a qualified teacher. The solution is revised on the basis of the gap between both the existing and the new means as

$$Differene\_Mean_i = r_i(M_{new} - T_F M_i \quad (15)$$

Where T_F is a teaching factor that affects the mean value to be changed, while r_i is a random number in the range [0 , 1] also The value of T_F can be 1 or 2, which is perhaps a heuristic step and determined arbitrarily with the same probability as T_F = Round[1 + rand(0, 1) {2 − 1}].

This discrepancy alters the existing solution to the following equation:

$$X_{new,i} = X_{old,i} + Difference\_Mean_i \quad (16)$$

Pupil's stage

Pupils may increase their knowledge in two ways: first, through feedback from their teacher, and second, from experiences between themselves. A student learns something different if the opposite student has more experience than he or she does. The pseudo-code of this method shall be described as Fig. 4

```
For i=1 : P_n    (17)
Pupil X_i and X_j selected randomly where i ≠ j
If f(X_i) < f(X_j)
X_new,i = X_old,i + r_i(X_i − X_j)   (18)
Else
X_new,i = X_old,i + r_i(X_j − X_i)   (19)
End if
End for
Accept X_new if it gives a better output
```

Figure 4. Pseudo-code of the pupil's stage

3.3 Hybrid TLBO and CSS algorithm

CSS and TLBO algorithms both have the initialization section in their algorithms. The key point within the Charged System Search (CSS) algorithm is to know the amount of CPs, the loaded memory size (CMS), the memory considering rate (CMCR) and therefore the pitch adjustment rate (PAR). Where we analyze CPs vectors and find the displacement of every node and stress in each member, then if the constraints are between the allowable limits, penalty sets adequate to zero. CSS is perfectly capable of exploration for the property. In order to have an improved algorithm, beside an excellent exploration there's a requirement to possess an accurate and efficient way of exploitation too. During this case, 'Teachers Phase' and 'Pupil Phase' of TLBO mixed with CSS to possess both great exploration and exploitation together. After identifying the right student individually, the TLBO algorithm teaches the remainder of the scholars through using their information to assist the category. Then, in next stage, students will aim to enhance their level by sharing knowledge. for every step, the trainer must change their position counting on the space to the measured class average. a private pupil often changes his/her place supported his/her gap from the category to a randomly chosen student. Through correcting the shortcomings of both algorithms by combining them, a replacement algorithm is developed that has the power to look sort of a CSS and therefore the ability to take advantage of sort of a TLBO, and productivity is far faster and more efficient than other methods. This new algorithm will lead to better results in a shorter time. The steps of the new hybrid algorithm are as follows:

Step 1: Initialize optimization problem and algorithm parameters required as number of CPs, CMS, CMCR and PAR, plus CP ranking in the second step of initialization and making the CM as population of students.

Step 2: Searching for a better CP in each section and choose it as a teacher, mentioned in Eq. (15) 'teachers' phase'.

Step 3: Adopt (Teacher CP) on the basis of best approach.

Step 4: Choose two random CPs (X_i and X_j as Eq. (17)) in the range of teacher CP as Eq. (20).

Step 5: Choose the best of X_i or X_j and apply it in CM

$$\begin{gathered} X_i, X_j \in B \\ Teacher\ CP - \varepsilon < B < Teacher\ CP + \varepsilon \end{gathered} \tag{20}$$

where X_i and X_j are best student CPs randomly selected from the set of B that is a collection of CPs equal or better than teacher CP. The flowchart of the hybrid algorithm is shown in Fig. 5.

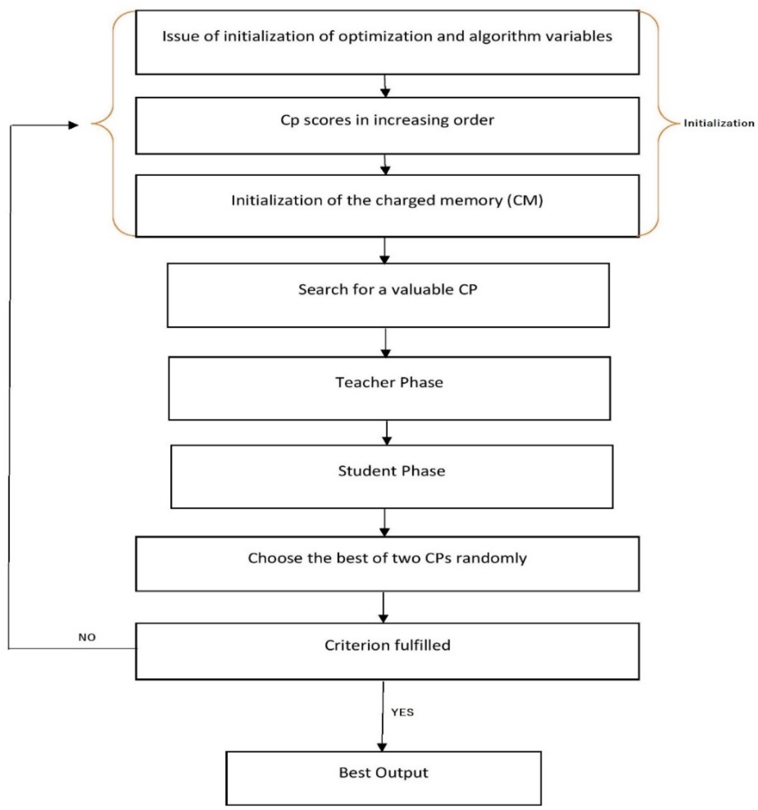
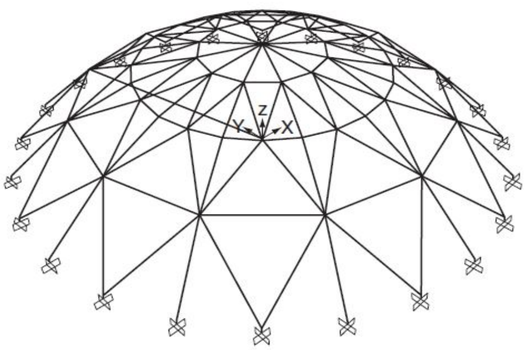


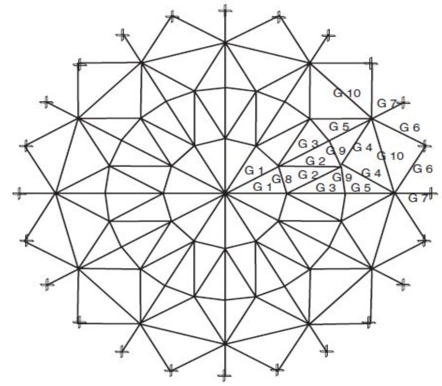
Figure 5. The flowchart of the hybrid algorithm

4. DESIGN EXAMPLES

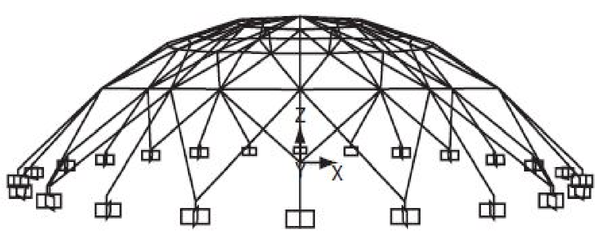
The behavior of the domes is non - linear due to the change in geometry under external loads. Existence of geometric non-linearity requires a push research and analysis. Additionally, an overall stability test is required during the study to make sure that the system does not actually lose its payload capacity due to instability [1]. Information of the nonlinear stiffness matrix of the space member are given in Majid [27] and Ekhande et al. [28] Furthermore, geometric non - linearity is also included in this analysis in order to provide a practical treatment of the dome.



(a) 3D view

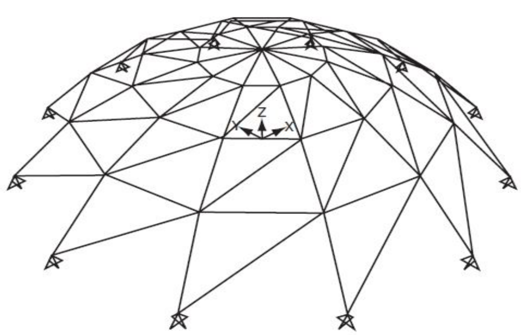


(b) Plan

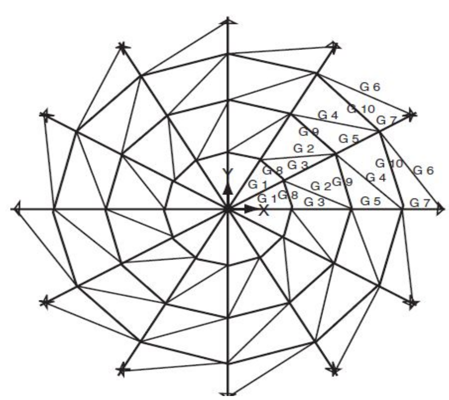


(c) Elevation view

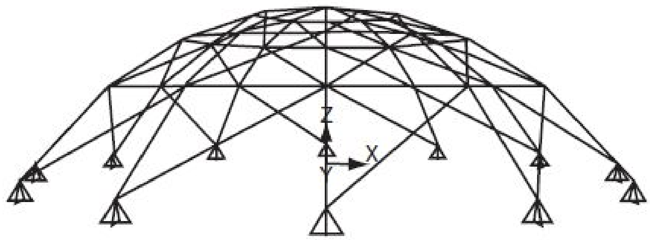
Figure 6. A four ring network dome



(a) 3D view

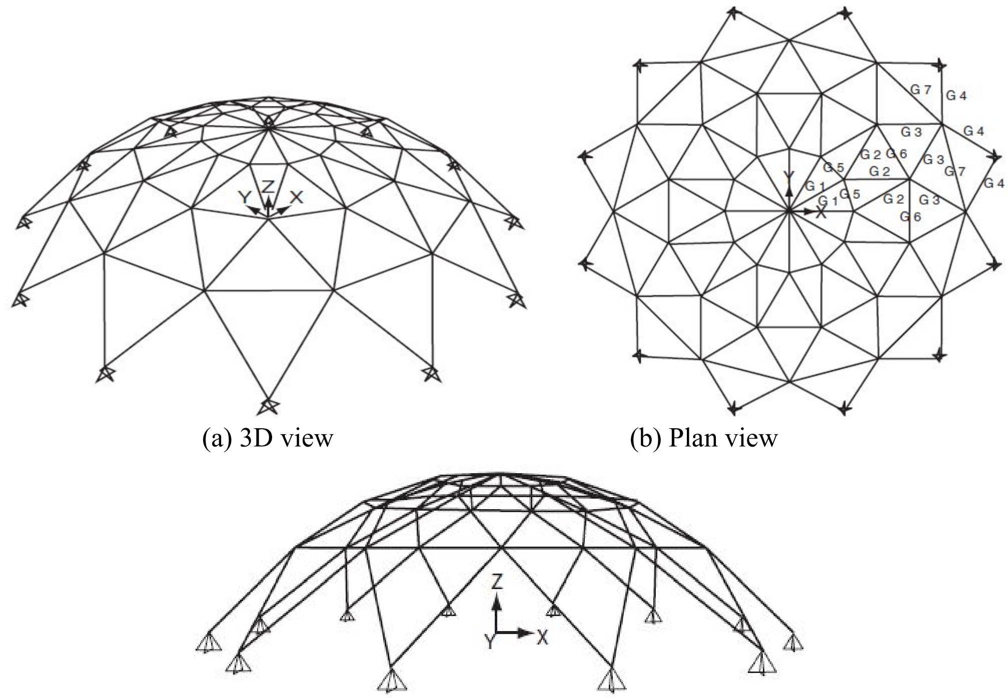


(b) Plan view



(c) Elevation view
Figure 7. A four-ring Schwedler dome

The network, the Schwedler and therefore the Lamella domes are being studied here. Network domes with rib, diagonal and ring elements as shown in Fig. 6. The gaps between the rings on the meridian line of those domes are usually rendered to be an equivalent. There are 12 joints on the odd rings, and therefore the same rings have 24 nodes. The first joint of the first ring is on the circumference of the dome, which correlates with the x-axis, and all the first joints of the rings are situated at the points of intersection of such rings and the x-axis. The Schwedler domes, one among the foremost common sorts of braced domes, consists of meridional ribs connected to a series of horizontal polygonal rings. In plan to stiffen the resulting dome, each trapezium formed by intersecting meridional ribs with horizontal rings is separated into two triangles by inserting a diagonal part. As shown in Fig. 7 for a standard Schwedler dome design, the amount of nodes for the Schwedler domes for every ring is taken into account to be constant and is adequate to 12 during this article. The gaps between the dome rings on the meridian line are generally of equal length. The Lamella dome includes diagonals extending from the crown right down to the equator of the dome, both clockwise and anticlockwise directions, and has horizontal rings, but has no meridian ribs. Similar to the Schwedler domes, the amount of nodes in each ring is taken as 12, whereas, in contrast to the 2 previous kinds, just the primary joints of the odd rings are situated at the points of intersection of the ring and therefore the x-axis, and therefore the first nodes of the evenly numbered rings are achieved by an anti-clockwise rotation of the nodes along the z-axis by 36 °. Fig. 8 illustrates a typical dome of lamella. The gathering of members is meant in such how that the rib members for every consecutive pair of rings relate to the same group, the diagonal members relate to an equivalent group, and therefore the members on each ring form a separate group. The overall number of groups for the network and the Schwedler domes is therefore equal to $3N_r - 2$ of lamella domes, this number is $2N_r - 1$, because there are no meridian ribs present. The group of various individuals is being used as the number of agents for all algorithms. Due to the continuous nature of the algorithms, a rounding function is used to develop separable or integer values.



(a) 3D view (b) Plan view

(c) Elevation view

Figure 8. A four-ring Lamella dome

The first example is a network dome. The histories of the best-run for different technique are shown in Fig. 9, and therefore the minimum results obtained by the algorithms are shown in Table 1. The optimum design for this dome is obtained using new method. The weight of the hybrid algorithm design is equal to 6,540 kg which is 8.68% and 5.96% lighter than the designs obtained by the TLBO and the CSS, respectively.

The Schwedler dome is that the second example. The lightest weight design is attained by the new algorithm, and therefore the optimum result obtained by the charged system search algorithm is that the runner-up as compared with the TLBO. The simplest weight of the Schwedler dome is adequate to 4,950 kg which is 13.21% and 11.96% lighter than the opposite designs. Table 2 summarizes the obtained optimum results. Similar the previous example, the new method has the fastest convergence rate and the TLBO has the slowest one as shown in Fig. 10.

The last structure investigated in this study is a lamella dome. The optimum design is obtained by the hybrid algorithm as 5777 kg. The CSS found the second weight which is 2.87% heavier than the results of the new method. Almost like the pervious examples, TLBO provide the heaviest result. Table 3 and Fig. 11 show the simplest results with the corresponding weight and therefore the convergence history of those algorithms for the lamella domes. When the number of rings increases, the load of all kinds of domes increase and thus to possess an optimal dome weight, the amount of rings should be chosen as small as possible. Within the case studies described above, the optimum number of rings obtained by algorithms is three.

Table 1: Optimum designs of the network dome

Network dome			
	Optimum sections		
Group Number	CSS	TLBO	New Method
1	PIPST (8)	PIPST (8)	PIPST (8)
2	PIPST (3)	PIPST (3)	PIPST (3)
3	PIPST (3)	PIPEST (31/2)	PIPST (31/2)
4	PIPST (3)	PIPST (31/2)	PIPST (21/2)
5	PIPST (31/2)	PIPST (31/2)	PIPST (3)
6	PIPEST (5)	PIPDEST (4)	PIPEST (5)
7	PIPDEST (2)	PIPST (4)	PIPST (31/2)
Height(m)	6.5	5.5	6.25
Max(δ_i)*(mm)*	27.44	27.13	27.95
Max Strength Ratio	0.93	0.96	0.96
Weight (kg)	6,955	7,162	6,540

Table 2: Optimum designs of the Schwedler dome

Schwedler dome			
	Optimum sections		
Group Number	CSS	TLBO	New Method
1	PIPDEST (4)	PIPDEST (5)	PIPST (8)
2	PIPEST (1/2)	PIPST (1/2)	PIPST (1/2)
3	PIPST (4)	PIPST (4)	PIPST (4)
4	PIPST (1/2)	PIPEST (1/2)	PIPST (3/4)
5	PIPST (4)	PIPST (4)	PIPST (31/2)
6	PIPEST (5)	PIPEST (5)	PIPDEST (4)
7	PIPST (6)	PIPST (4)	PIPEST (3)
Height (m)	7.75	7.25	6.00
Max(δ_i) *(mm)*	28.11	26.64	28.00
Max Strength Ratio	1.00	0.96	0.97
Weight (kg)	5,623	5,704	4,950

Table 3: Optimum designs of the lamella dome

Lamella Dome			
	Optimum sections		
Group Number	CSS	TLBO	Present Method
1	PIPST (8)	PIPST (8)	PIPST (3)
2	PIPST (31/2)	PIPST (3)	PIPST (3)
3	PIPST (3)	PIPEST (3)	PIPST (3)

4	PIPDEST (4)	PIPEST (6)	PIPDEST (4)
5	PIPST (31/2)	PIPEST (31/2)	PIPST (31/2)
Height (m)	6.25	5.75	6.50
Max (δ_i) *(mm)*	27.85	27.35	27.78
Max Strength Ratio	0.989	0.932	1.00
Weight (kg)	5,948	6,404	5,777

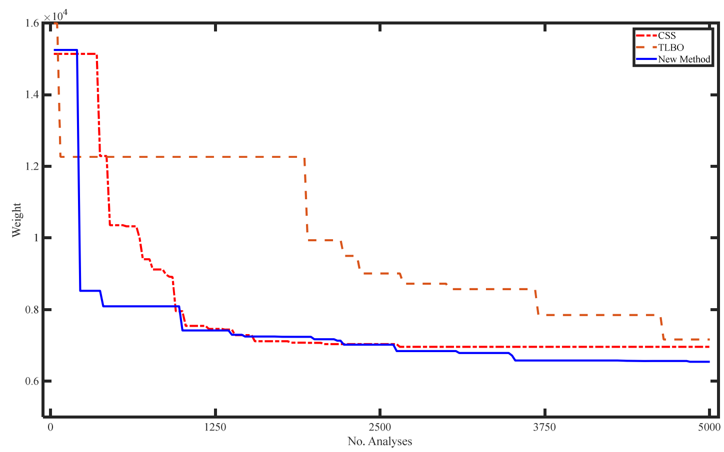


Figure 9. The convergence histories for the network dome

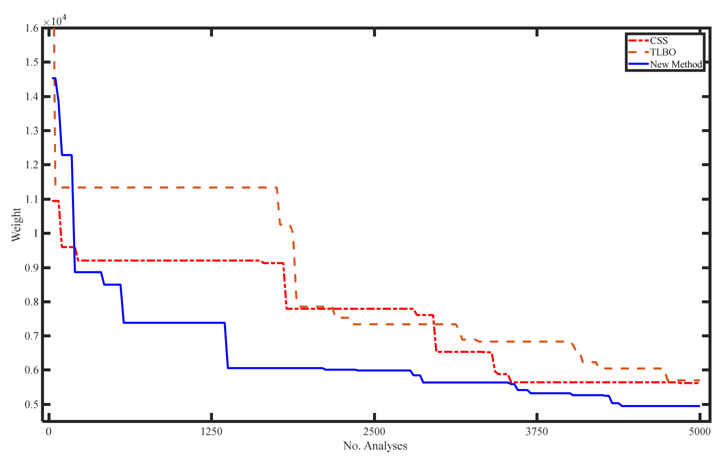


Figure 10. The convergence histories for the Schwedler dome

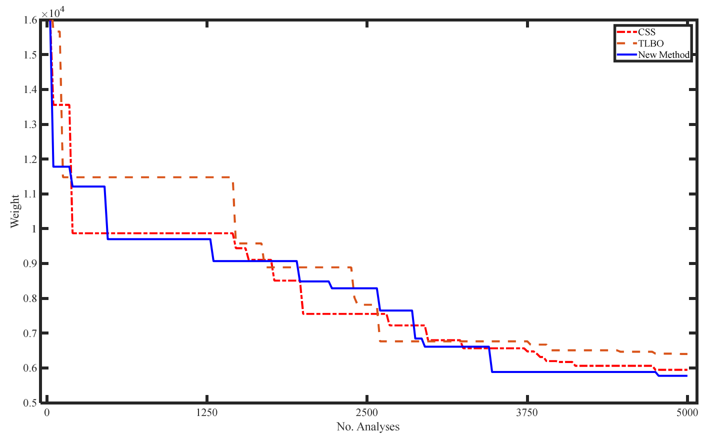


Figure 11. The convergence histories for the Lamella dome

5. CONCLUDING REMARKS

For dome structures due to the existence of large search spaces, an outsized number of optimization constraints should be handled, considering the fact that these issues are very difficult due to the computational complexity of the dome structure analysis. Recently, various researchers have used combined optimization algorithms to overcome these difficulties. The present hybrid algorithm overcomes the optimization problems of dome structures, by combining the CSS and TLBO algorithms to eliminate the operational weakness of the CSS algorithm by adding two 'Teachers phase' and 'Pupil phase' to obtain efficient results and to scale back computation time. In this paper, three types of dome designs are optimized by three algorithms, consisting of CSS, TLBO and the new hybrid method. The analysis results during this comparative process indicate that the new method provides a better and simpler design in less time than the standard algorithms.

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