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PARAMETRIC STUDY: COST OPTIMIZATION OF NON-PRISMATIC REINFORCED CONCRETE BOX GIRDER BRIDGES WITH DIFFERENT NUMBER OF CELLS

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ABSTRACT

In this paper the parametric study is carried out to investigate the effect of number of cells in optimal cost of the non-prismatic reinforced concrete (RC) box girder bridges. The variables are geometry of cross section, tapered length, concrete strength and reinforcement of the box girders and slabs that are obtained using ECBO metaheuristic algorithm. The design is based on AASHTO standard specification. The constraints are the bending and shear strength, geometric limitations and superstructure deflection. The link of CSiBridge and MATLAB software are used for the optimization process. The methodology carried out for two-cell, three-cell and four-cell box girder bridges. The results show that the total cost of the concrete, bars and formwork for two-cell box girder is less than those of the three- and four-cell box girder bridges.

Keywords: optimal cost; RC box girder bridge; non-prismatic; number of cells; enhanced colliding bodies optimization (ECBO) algorithm.

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1. INTRODUCTION

Structural engineers try to design structures that are economical and have sufficient strength. The use of traditional (trial and error) method in the design of structures is not sufficient to meets both economic and safety criteria simultaneously. Recently developed stochastic search algorithms that have made it possible to move from the traditional (trial-and-error) design to optimal design of problems. Optimal design of RC frames is more complex. Because they have large number of variables and in optimizing RC frames, the cost of three

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different components including concrete, steel and formwork must be considered. In recent decades, the optimal design of reinforced concrete frames has attracted the attention of many researchers with the objective of cost or CO_2 emissions [1-8]. Studies have also been conducted on the optimal design of bridges. Martí et al. [9] describes one approach to optimize the economic cost of prestressed concrete precast road bridges by hybrid simulated annealing. Where the bridges are formed with double U-shaped cross-sections and RC slabs. Yepes et al. [10] optimize the cost and CO_2 emission of precast–prestressed concrete road bridges with a double U-shape cross-section. They used the hybrid glowworm swarm algorithm to obtain the optimal variables. Martínez et al. [11] developed a framework for the optimal design of RC tall bridge piers with hollow rectangular sections with the ant colony optimization algorithm. Kaveh et al. [12] according to the specifications of AASHTO 2002 standard, optimize the cost of post-tensioned concrete box girder of single span bridges. In their study the problem is formed by 17 design variables and 135 constraints and the optimal variable obtained with PSO, CBO and MCBO algorithms. In another study, Kaveh et al. [13] used three metaheuristic algorithms including CBO, VPS and ECBO to optimize the steel-concrete composite I-girder bridges. Pedro et al. [14] optimized the cost of steel-concrete composite I-girder bridges based on an efficient two-stage optimization approach. Yepes et al. [15] proposed a methodology to minimize the cost of the post-tensioned concrete box-girder pedestrian deck based on the Spanish code. In another study, García-Segura et al. [16] minimized the CO_2 emissions, cost and overall safety factor of post-tensioned concrete road bridges. Penadés Plà et al.[17] used a robust design optimization method to design a continuous prestressed concrete box girder pedestrian bridge.

From a review of the literature, it can be concluded that the effect of number of cells on cost of bridge has not yet been investigated. This research presents a parametric study to investigate the effect of number of cells on the optimal cost of non-prismatic reinforced concrete box girder bridge. The methodology carried out for two cell, three cell and four cell box girder bridge. The variables are geometry of cross section, tapered length, concrete strength and reinforcement of box girders and slabs. The design is based on AASHTO 2002 standard specification. The link of CSiBridge and Matlab software has been used for optimization.

2. OPTIMIZATION ALGORITHM

Enhanced Colliding Bodies Optimization (ECBO) [18] algorithm was used to optimize the problem in this study. The colliding bodies optimization (CBO) algorithm [19] and ECBO algorithm are based on the physical laws governing the collision between objects. Where the momentum before the collision is equal to the sum of the momentum after the collision. The ECBO algorithm uses Memory and the *Pro* parameters to escape from local optima and to increase the convergence speed of the CBO algorithm. Memory that saves a number of the best solutions in each iteration and substitute them with the current worst objects. Using *Pro* parameter, one component of the *i*th Colliding Body (CB) is regenerated randomly in each iteration. This parameter is in the range of (0, 1). Each CB has a specified mass defined as:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \quad k = 1,2,\dots,n \tag{1}$$

where $fit(i)$ presents the objective function value of the ith colliding body and n is the number of populations. In order to select the pairs for collision, the objects are sorted according to their weights in a decreasing order and divided in to two equal groups: 1) stationary and 2) moving objects. Fig. 1. Moving objects collide with stationary objects to improve their positions and push stationary objects toward better positions. Using the velocity before and after collision, the position of the objects was updated. Further explanations on CBO, ECBO and other metaheuristic algorithms and their applications can be found in Kaveh [20, 21]. Matlab codes for efficient metaheuristics are provided by Kaveh and Bakhshpoori [22], and real sized structures are optimized in Kaveh and Ilchi Ghazaan [23].

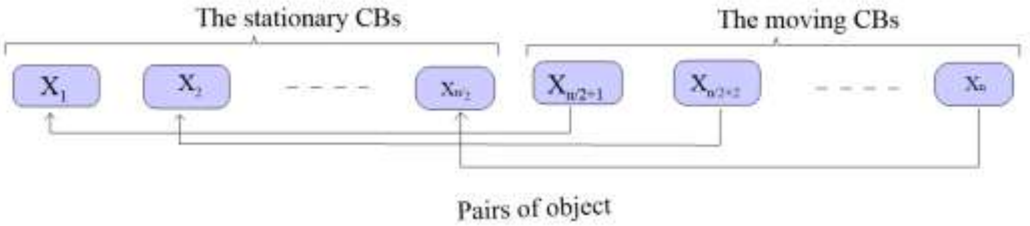

Figure 1. The pairs of objects for collision

3. FORMULATION FOR DESIGN OF BRIDGE

3.1 Loads

The bridge must be designed to withstand dead and moving loads. According to AASHTO 2002 [24], the combination of dead and live loads (Eq. (2)) is used for superstructure loading. Dead loads (DL) include the weight of girders, slabs and the weight of asphalt. According to the articles 3.7 of AASHTO 2002, H20-44 and HS20-44 are considered as Live load (LL). The width of the deck is 9.2 meters and two traffic lanes with a width of 3.6 meter have been used.

Combination load is:

$$1.3DL + 2.17LL \tag{2}$$

In the live load the dynamic effects are calculated as:

$$MI = 1 + \frac{50}{3.28L + 125} \leq 1.3 \tag{3}$$

where L is the length of span in meter.

3.2 Design variables

The variables in this study are concrete strength, geometry of the cross section, tapered length, reinforcement of box girders and slabs. Design variables and parameters are tabulated in Table 1. A typical geometry cross-section of the bridge with some of the variables is shown in Fig. 2.

Table 1: Design variables and parameters

No.	Variable	Symbol	Step	Constraints
1	Concrete strength (kg/cm^2)	f_c'	50	$250 \leq f_c' \leq 500$
2	Girder depth (m)	h1, h3, h5	0.25	$1 \leq h \leq 2.5$
3	Girder depth in support (m)	h2, h4	0.25	$1.5 \leq h \leq 3$
4	Top slab thickness (cm)	T_t	1	$18 \leq T_t \leq 35$
5	Bottom slab thickness (cm)	T_b	1	$17 \leq T_b \leq 30$
6	End thickness of cantilever (cm)	T_c	1	$18 \leq T_c \leq 30$
7	Initial thickness of cantilever (cm)	T_s	2	$20 \leq T_s \leq 50$
8	Length of cantilever (m)	L_c	0.25	$1 \leq L_c \leq 2$
9	Web thickness in intermediate cell (cm)	T_{W3}	2	$25 \leq T_{W1} \leq 50$
10	web thickness in outside cell (cm)	T_{W1}	2	$30 \leq T_{W1} \leq 70$
11	Diameter of reinforcement perpendicular to traffic in top slab	d_1	1	$\#3 \leq d_1 \leq \#11$
12	Number of reinforcement perpendicular to traffic in top slab	n_1	1	$2 \leq n_1 \leq 15$
13	Diameter of reinforcement perpendicular to traffic in cantilever	d_2	1	$\#3 \leq d_2 \leq \#11$
14	Number of reinforcement perpendicular to traffic in cantilever	n_2	1	$2 \leq n_2 \leq 15$
15	Number of bars in moment capacity for sections	nlt, nlb	2	$2 \leq n_{lt}, n_{lb} \leq 30$
16	Diameter of bars in in moment capacity for sections	dlt	constant	#8
17	Diameter of shear bars (mm)		constant	12
18	Tapered length (TLR) (m)	TLR	1	$3 \leq TLR \leq 7$
19	t1=t2=t3=t4=t5=t6=t7=t8 (mm)		constant	150
20	Number of cells		constant	2,3,4

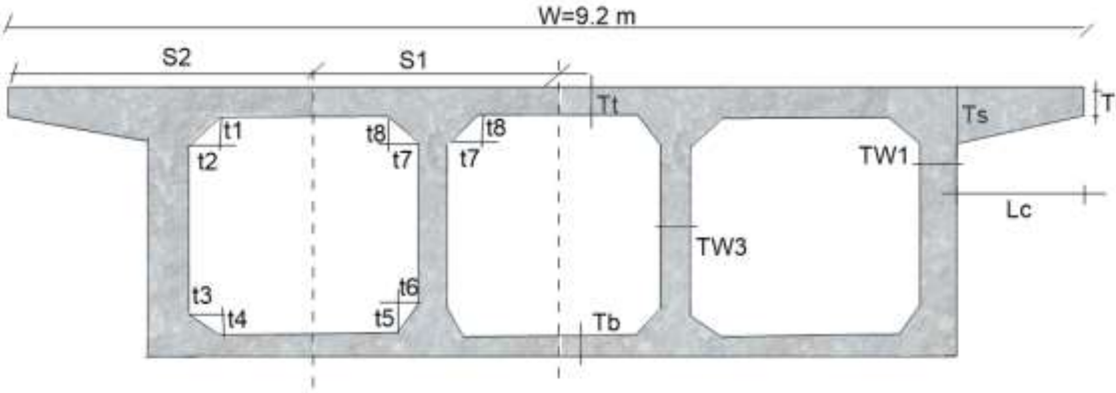

Figure 2. Geometry of superstructures

3.3 Design checks

The design of reinforced concrete slabs and girders is based on AASHTO 2002 specification. In all sections, flexure strength, shear strength, geometry constraints and superstructure deflection are controlled. Also, the main and distribution reinforcement of slabs, longitudinal skin reinforcement according AASHTO code are calculated.

3.4 Objective function

To find the optimal design while satisfying the constraints, the formulation is shown as Eq. (4). The objective function in this study is to minimize the cost of the materials that contain volume of concrete, weight of reinforcement and area of formwork in the RC bridge.

$$\begin{array}{cc} \text{Find} & \{X\} = [x_1 , x_2 , \dots , x_n] \\ \text{to minimize} & f(\{X\}) = V_c . C_c + C_s . \gamma_s . A_s . L_s + C_f . A_f \\ \text{subjected to} & g_j(x) \leq 0, \quad j = 1 \; to \; m \\ \text{where} & x_{min} \leq \text{x} \leq x_{max} \end{array} \tag{4}$$

Where $f(\{X\})$ presents the cost of the superstructure bridge. C_c , C_s and C_f are the unit rate of concrete, bars and formwork, respectively. Their values for the objective function are given in Table 2. V_c is the volume of concrete, that is extract from the CSiBridge software; γ_s is unit weight of bars that is 7850 kg/m^3; A_s and L_s are the area and length of bars, respectively; A_f is area of formwork. {X} is the vector containing the design variables; *n* is the number of variables; x_{min} and x_{max} are the lower and upper bounds of the design variable; $g_j(x)$ denotes design constraints, and *m* is the number of the constraints.

In order to handling the design constraints a penalty function is used. Using penalty functions the constrained problem can be transformed into unconstrained problem as:

$$f_p(x) = f \times (1 + \sum_{i=1}^{m} \max(0, g_i(x)))^{\,k} \tag{5}$$

Where f_p represents the penalized objective function, f denotes the value of the objective function, and k denotes a penalty exponent, where $k=1.6$ is considered in this study.

Table 2: Unit prices of the cost function [16]

Item	Description	**Cost (€)**
C_s	Kg of Steel B-500	1.16
C_c	m^3 of Concrete (25 MPa)	95.05
	m^3 of Concrete (30 MPa)	99.81
	m^3 of Concrete (35 MPa)	104.57
	m^3 of Concrete (40 MPa)	109.33
	m^3 of Concrete (45 MPa)	114.10
	m^3 of Concrete (50 MPa)	118.87
C_f	m^2 of Formwork	33.81

3.5 Methodology for optimization

The link of CSiBridge and Matlab softwares have been used in the optimization process. MATLAB interacts with CSiBridge through its Application Programming Interface (API). In which MATLAB is used for handled the optimization algorithm and verification the AASHTO standard specification. CSiBridge is used for finite element analysis. Shell elements with sub mesh size of 1.2 m and maximum segment length of 1 m are used in superstructure modeling. In initial, the bridge model is created in CSiBridge software and the *$br* file is saved, which will be later imported by CSiBridge to analysis the model. This document is used to define and update the variables. The variable is updated via an optimization algorithm. The results of model are extracted using OAPI functions.

4. NUMERICAL EXAMPLE

In this section, the optimization of a box girder reinforced concrete bridge with three spans with lengths of 15, 26 and 20 meters is presented. Optimization is performed for different number of cells. The objective function is economic cost. In order to design and control the constraints, the superstructure is divided in to 31 parts (section cut) and 19 section (Fig. 3). Section cuts and related variables are shown in Table 3.

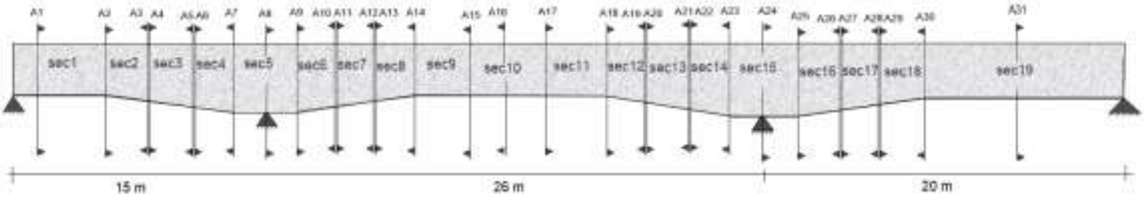

Figure 3. Bridge division for design

The variables listed in Table 1 are the same in all different sections, except for the items listed in Table 3. In this table, *htlr* is obtained for non-prismatic sections by interpolation.

Table 3: Sections and related variables

Section cut	**Depth of girders (h)**	**Number of longitudinal bars (top)**	**Number of longitudinal bars (bottom)**	**Space of shear bar (S)**
A1	h1	nLt1	nLb1	S1
A2	h1	nLt2	nLb2	S2
A3	htlr1	nLt2	nLb2	S2
A4	htlr1	nLt3	nLb3	S3
A5	htlr2	nLt3	nLb3	S3
A6	htlr2	nLt4	nLb4	S4
A7	h2	nLt4	nLb4	S4
A8	h2	nLt5	nLb5	S5
A9	h2	nLt6	nLb6	S6
A10	htlr3	nLt6	nLb6	S6
A11	htlr3	nLt7	nLb7	S7
A12	htlr4	nLt7	nLb7	S7
A13	htrl4	nLt8	nLb8	S8
A14	h3	nLt8	nLb8	S8
A15	h3	nLt9	nLb9	S9
A16	h3	nLt10	nLb10	S10
A17	h3	nLt11	nLb11	S11
A18	h3	nLt12	nLb12	S12
A19	htlr5	nLt12	nLb12	S12
A20	htlr5	nLt13	nLb13	S13
A21	htlr6	nLt13	nLb13	S13
A22	htlr6	nLt14	nLb14	S14
A23	h4	nLt14	nLb14	S14
A24	h4	nLt15	nLb15	S15
A25	h4	nLt16	nLb16	S16
A26	htlr7	nLt16	nLb16	S16
A27	htlr7	nLt17	nLb17	S17
A28	htlr8	nLt17	nLb17	S17
A29	htrl18	nLt18	nLb18	S18
A30	h5	nLt18	nLb18	S18
A31	h5	nLt19	nLb19	S19

4.1 Bridge with 2 cells

Fig. 4 shows the cross section of the bridge with two cells. The optimal results listed in Table 4 and Table 5. Where the best cost is 128937.43 euro. The volume of concrete for bridge with two cells is 323.6564 m^3, the total weight of the bars in slabs and girders are 37288.8 kg and the area of formwork is 1487.6 m^2. The cost of concrete is 35385 euro. The cost of reinforcements is 43255 euro. The cost of formwork is 50297 euro.

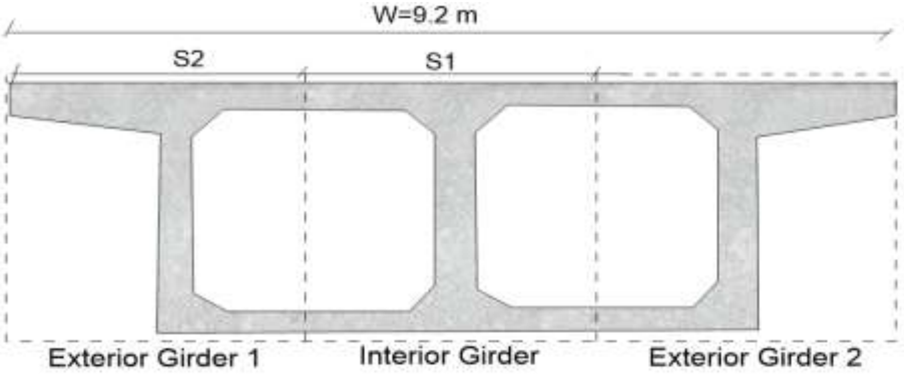

Figure 4. Cross section of deck for bridge with 2 cells

Table 4: Optimum longitudinal bars, depth of girders and also space of shear bars for bridge with 2 cells

Section	**Girders**						**Depth (m)**	
	Exterior Girders			**Interior Girders**			**h node i**	**h node j**
	nlt (top)	**nlb (bottom)**	**S (m)**	**nlt (top)**	**nlb (bottom)**	**S (m)**		
Sec 1	6	6	0.4	6	8	0.3	1.25	1.25
Sec 2	8	6	0.4	6	8	0.3	1.25	1.65
Sec 3	10	2	0.4	8	2	0.3	1.65	2.05
Sec 4	12	2	0.6	8	6	0.3	2.05	2.25
Sec 5	12	2	0.4	12	2	0.3	2.25	2.25
Sec 6	12	2	0.4	10	4	0.3	2.25	2.1875
Sec 7	12	4	0.3	8	6	0.3	2.1875	2.125
Sec 8	12	10	0.3	8	10	0.3	2.125	2
Sec 9	10	10	0.5	8	10	0.4	2	2
Sec 10	2	12	0.6	4	12	0.6	2	2
Sec 11	10	10	0.4	8	10	0.3	2	2
Sec 12	10	10	0.3	8	8	0.2	2	2.125
Sec 13	12	4	0.3	14	4	0.2	2.125	2.1875
Sec 14	12	2	0.3	12	4	0.3	2.1875	2.25
Sec1 5	14	2	0.3	14	2	0.3	2.25	2.25
Sec 16	12	2	0.3	12	2	0.2	2.25	2
Sec 17	10	8	0.3	8	2	0.2	2	1.75
Sec 18	12	10	0.3	6	10	0.2	1.75	1.5
Sec 19	8	10	0.3	6	12	0.3	1.5	1.5

Table 5: Optimum result for bridge with 2 cells

Optimum variable	f_c' (kg/cm2)	400
	T_t (cm)	22
	T_b (cm)	17
	T_c (cm)	25
	T_s (cm)	34
	L_c (m)	1.75

	T_{W3} (cm)	27
	T_{W1} (cm)	38
	$Top\ slab\ reinforcement/m\ ;(n_1, d_1)$	2#8
	$Cantilever\ slab\ reinforcement\ /m;\ (n_2, d_2)$	8#4
	TLR1 (span1) (m)	5
	TLR2 (span2) (m)	4
	TLR3 (span3) (m)	6
Best solution	Cost 128937.43 €	

4.2 Bridge with 3 cells

The cross section of the bridge with 3 cells is shown in Fig. 5. The results are listed in Table 6 and Table 7. Where the best cost is 131271.2 euro. The volume of concrete for bridge with 3 cells is 333.64 m^3, the total weight of the bars in slabs and girders are 34908 kg and the area of formwork is 1653 m^2. The cost of concrete is 34889 euro. The cost of reinforcements is 40493 euro. The cost of formwork is 55889 euro.

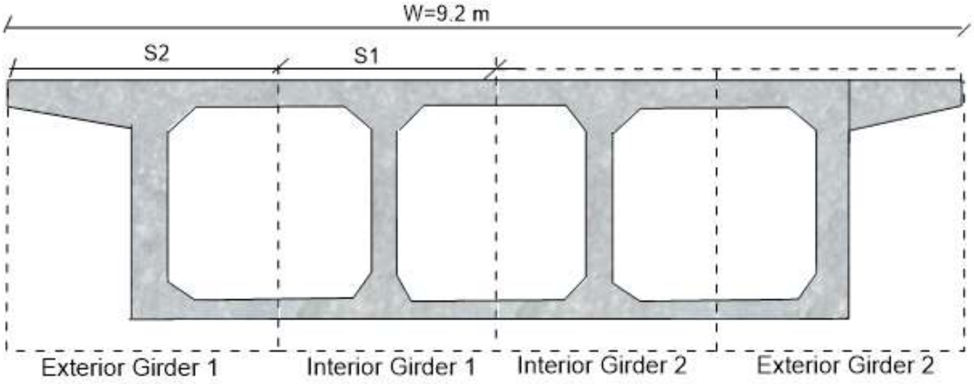

Figure 5. Cross section of deck for bridge with 3 cells

Table 6: Optimum longitudinal bars, depth of girders and also space of shear bars for bridge with 3 cells

	Girders						Depth (m)	
Section	**Exterior Girders**			**Interior Girders**			**h node i**	**h node j**
	nlt (top)	**nlb (bottom)**	**S(m)**	**nlt (top)**	**nlb (bottom)**	**S(m)**		
Sec 1	6	6	0.5	6	6	0.4	1.5	1.5
Sec 2	8	2	0.5	6	2	0.4	1.5	1.75
Sec 3	10	2	0.6	8	2	0.4	1.75	2
Sec 4	10	2	0.6	8	2	0.5	2	2.25
Sec 5	10	2	0.6	8	2	0.5	2.25	2.25
Sec 6	10	2	0.6	8	2	0.4	2.25	2.0625
Sec 7	10	2	0.5	6	2	0.3	2.06	1.875
Sec 8	8	6	0.4	6	6	0.3	1.875	1.5

Sec 9	6	8	0.4	6	8	0.3	1.5	1.5
Sec 10	2	8	0.6	2	10	0.6	1.5	1.5
Sec 11	6	8	0.3	6	8	0.3	1.5	1.5
Sec 12	8	6	0.3	6	6	0.3	1.5	1.875
Sec 13	10	2	0.4	8	2	0.3	1.875	2.06
Sec 14	10	2	0.5	8	2	0.4	2.06	2.25
Sec1 5	10	2	0.6	10	2	0.4	2.25	2.25
Sec 16	10	2	0.6	8	2	0.4	2.25	2.1
Sec 17	12	2	0.5	8	2	0.3	2.1	1.8
Sec 18	8	6	0.4	6	6	0.3	1.8	1.5
Sec 19	6	8	0.4	6	8	0.3	1.5	1.5

Table 7: Optimum result for bridge with 3 cells

Optimum variable	f_c' (kg/cm2)	350
	$T_t\ (cm)$	22
	$T_b\ (cm)$	17
	$T_c\ (cm)$	18
	$T_s\ (cm)$	26
	$L_c\ (m)$	1.25
	$T_{W3}\ (cm)$	25
	$T_{W1}\ (cm)$	36
	$Top\ slab\ reinforcement/m\ ; (n_1, d_1)$	2#7
	$Cantilever\ slab\ reinforcement\ /m;\ (n_2, d_2)$	2#7
	TLR1 (span1) (m)	3
	TLR2 (span2) (m)	4
	TLR3 (span3) (m)	5
Best solution	Cost 131271.2 €	

4.3 Bridge with 4 cells

Fig. 6 shows the cross section of the bridge with 4 cells. The results are listed in Table 8 and Table 9. Where the best cost is 132041.3 euro. The volume of concrete for the bridge with 4 cells is 349.89 m^3 and the total weight of the bars in slabs and girders are 35553.6 kg. the area of formwork is 1652.6 m^2. The cost of concrete is 34923 euro. The cost of reinforcements is 41242 euro. The cost of formwork is 55876 euro.

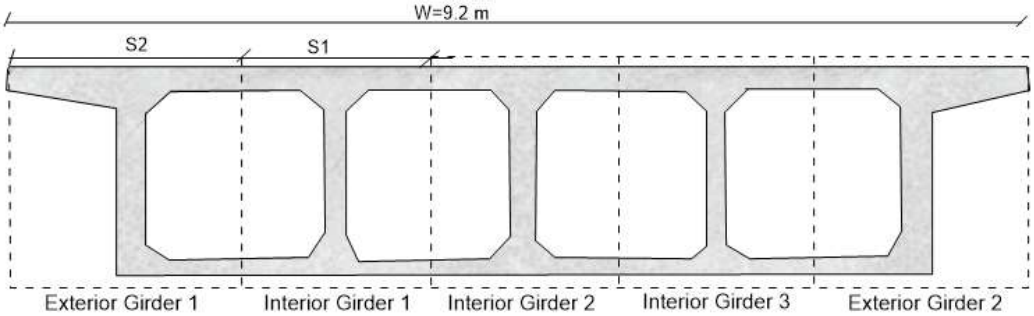

Figure 6. Cross section of deck with 4 cells

Table 8: Optimum longitudinal bars, depth of girders and also space of shear bars for bridge with 4 cells

	Girders						**Depth (m)**	
Section	**Exterior Girders**			**Interior Girders**			**h node i**	**h node j**
	nlt (top)	**nlb (bottom)**	**S(m)**	**nlt (top)**	**nlb (bottom)**	**S(m)**		
Sec 1	6	6	0.3	6	6	0.3	1	1
Sec 2	6	2	0.3	6	4	0.3	1	1.5
Sec 3	8	2	0.6	6	2	0.4	1.5	1.75
Sec 4	8	2	0.6	8	2	0.4	1.75	2
Sec 5	10	2	0.6	8	2	0.5	2	2
Sec 6	8	4	0.5	6	2	0.4	2	1.85
Sec 7	8	4	0.4	10	2	0.4	1.85	1.55
Sec 8	6	6	0.4	6	4	0.4	1.55	1.25
Sec 9	8	8	0.4	4	10	0.4	1.25	1.25
Sec 10	2	8	0.5	2	8	0.5	1.25	1.25
Sec 11	6	8	0.3	4	8	0.3	1.25	1.25
Sec 12	6	6	0.3	8	4	0.3	1.25	1.55
Sec 13	8	2	0.4	8	2	0.3	1.55	1.85
Sec 14	8	2	0.4	10	2	0.3	1.85	2
Sec1 5	10	2	0.5	10	2	0.4	2	2
Sec 16	8	4	0.4	10	2	0.3	2	1.833
Sec 17	8	2	0.4	8	4	0.3	1.83	1.66
Sec 18	6	4	0.3	8	2	0.3	1.66	1.5
Sec 19	8	8	0.3	6	8	0.3	1.5	1.5

Table 9: Optimum result for bridge with 4 cells

Optimum variable	f_c' (kg/cm2)	300
	$T_t\ (cm)$	22
	$T_b\ (cm)$	17
	$T_c\ (cm)$	18
	$T_s\ (cm)$	36
	$L_c\ (m)$	1.25
	$T_{W3}\ (cm)$	27

	$T_{W1}\ (cm)$	34
	$Top\ slab\ reinforcement/m\ ;(n_1, d_1)$	5#4
	$Cantilever\ slab\ reinforcement\ /m;\ (n_2, d_2)$	5#4
	TLR1 (span1) (m)	4
	TLR2 (span2) (m)	5
	TLR3 (span3) (m)	3
Best solution	Cost 132041.3 €	

Comparative results for the cost of bridge with two-cell, three-cell and four-cell are given in Table 10.

Table 10: Comparative results for the cost of bridge with different cells

Number of cells	**Optimal cost (€)**				
	Concrete	**Reinforcement**	**Formwork**	**Concrete + Reinforcement**	**Total**
2	35385	43255	50297	78640	128937.43
3	34889	40493	55889	75382	131271.2
4	34923	41242	55876	76165	132041.3

5. CONCLUDING REMARKS

This research presents a parametric study to investigate the effect of number of cells on the optimal cost of non-prismatic reinforced concrete box girder bridge. Optimization performed for a three-span bridge with 2, 3 and 4 cells. The variables are geometry, tapered length, concrete strength, reinforcement of box girders, reinforcement of slabs. The constraints are the bending strength, shear strength, deflection and geometric limits based on the AASHTO 2002 standard specification. A computer tool that is the link of CSiBridge and MATLAB softwares are utilized for the optimization process. Where CSiBridge software is used for finite element analysis. The check of AASHTO standard specification and optimization algorithm are handled in MATLAB software. Optimal results for bridges are obtained using the enhanced colliding bodies optimization algorithm. The results indicated the total cost of concrete and bars for three-cell box girder is less than of two-cell and four-cell box girder. On the other hand, due to the fact that as the number of cells increases, the amount of formwork used increases, therefore by considering the cost of the formwork, the total cost of concrete, bars and formwork for two-cell box girders is less than the other two.

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