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TRUSS LAYOUT AND SIZE OPTIMIZATION CONSIDERING DYNAMIC CONSTRAINTS UISNG WATER EVAPORATION OPTIMIZATION ALGORITHM

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ABSTRACT

Metaheuristics are considered the first choice in addressing structural optimization problems. One of the complicated structural optimization problems is the highly nonlinear dynamic truss shape and size optimization with multiple natural frequency constraints. On the other hand, natural frequency constraints are useful to control the responses of a dynamically exciting structure. In this regard, this study uses for the first time the water evaporation optimization (WEO) algorithm to address this problem. Four benchmark trusses are considered for experimental investigation of the WEO. Obtained results indicate the comparative performance of WEO to the best-known algorithms in this problem, high performance in comparison to those of different optimization techniques, and high performance in comparison to all algorithms in terms of robustness. The simulation results clearly show a good balance between the global and local exploration abilities of WEO and its potential robust efficiency for other complicated constrained engineering optimization problems.

Keywords: truss optimization; frequency constraints; metaheuristic algorithms; Water Evaporation Optimization.

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1. INTRODUCTION

Metaheuristics are today the first choice for researchers and practitioners in addressing various types of optimization problems. In this regard knowledge about these algorithms is expanding day by day and this development, in general, can be divided into two important

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branches: algorithmic and application aspects. The first one tries to improve metaheuristics [1 and 2] to have more better performance and the latter tries to adopt them for new optimization problems. Although the first branch is necessary to have efficient algorithms, the second one must also be considered simultaneously to give us comprehensive algorithm effectiveness. For the first branch novel algorithms have been invented or improved using different strategies. The second branch has started to grow interdisciplinary almost seriously since the beginning of this century. This growth coincides with the development of computer science in terms of software and hardware, which has enabled the scientific community to model and solve complex problems. Between the novel algorithms, one can refer to the charged system search, the ray optimization, the colliding bodies optimization, the teaching-learning-based optimization, the artificial bee colony algorithm, the harmony search, the grey wolf optimizer, the cuckoo search algorithm as the most well-known recent metaheuristics [3-10]. Between optimization problems along with combinatorial optimization problems, engineering problems are known as the most complex optimization problems because of their constrained and mixed continuous and discrete nature [11]. Structural optimization problems are one of the most well-known engineering optimization problems and among them, the single objective structural weight optimization is still challenging especially if the frequency constraints are included [12].

The water evaporation optimization (WEO) algorithm is one of the recent metaheuristics invented by Kaveh and Bakhshpoori [12] inspired by the evaporation process of a tiny amount of water molecules adhered on a solid surface with different wettability. WEO has been successfully applied on single objective structural weight optimization of 2D and 3D truss and frame structures [13-15]. Dynamic truss shape and size optimization with multiple natural frequency constraints is known as one of the highly nonlinear structural optimization problems. It has been studied since the 1980s with the paper of Bellagamba and Yang [16] and continuing with many other researchers using various metaheuristics [17-25]. An improved version of the particle swarm optimization named democratic PSO is used for truss layout and size optimization with frequency constraints in [17]. [18] applied colliding bodies optimization for truss optimization with multiple frequency constraints. Millan and Filho [19] developed a modified simulated annealing algorithm to address this problem. In another study, they presented a modified version of the social engineering optimizer and solved the truss optimization problem with natural frequency constraints [20]. Kaveh et al. addressed this problem for benchmark trusses [21 and 22] dome structures [23], cyclically symmetric trusses [24], large-scale dome-shaped trusses [25], and cyclically large-size braced steel domes [26]. The main reasons for this focus are: frequency constraints are highly non-linear, non-convex, and implicit concerning the design variables, natural frequencies of a structure provide useful information about the dynamic behavior of the system, and considering the simultaneous shape and sizing optimization,

This study aims to adopt the WEO for truss layout and size optimization problems considering dynamic constraints to increase its comprehensiveness and popularity. For this purpose, WEO experimented on four benchmark trusses taken from literature. Performance evaluation is made by comparing with other algorithms. Numerical results reveal that the WEO performs comparatively to other search techniques available in the literature.

The remaining sections of this paper are structured as follows. In Section 2 WEO algorithm is outlined and the problem is formulated. Section 3 applies WEO on the four benchmark trusses and compares its performance against other algorithms. Section 5 investigates the algorithm convergence behavior. Finally, the paper is concluded in Section 4.

2. FORMULATION OF THE OPTIMIZATION PROBLEM

In a frequency constraint truss layout and size optimization problem the aim is to minimize the weight of the structure while satisfying some constraints on natural frequencies. The design variables are considered to be the cross-sectional areas of the members and/or the coordinates of some nodes. Prescribing the truss topology and assuming it to be unchanged, the optimization problem can be stated as follows:

$$\begin{aligned}&\text{Find}\left\{\mathrm{V}\right\}=\left[v_i\right] \quad i=1,2,\ldots,k, \;\; v_{\min}\le v_k \le v_{\max},\\ &\text{To minimize:}\quad \mathrm{W}\left\{\mathrm{V}\right\}=\sum_{e=1}^{n} L_e \rho_e A_e, \quad \text{Subject to:}\end{aligned} \tag{1}$$

$$g_1\{\mathrm{V}\}: \omega_j^* - \omega_j \le 0 \quad \text{for some natural frequencies j} \tag{2}$$

$$g_2\{\mathrm{V}\}: \omega_{jj} - \omega_{jj}^* \le 0 \quad \text{for some natural frequencies jj} \tag{3}$$

where *{V}* is the set of design variables; *k* is the number of independent design variables, v_i, including either a shape or sizing variable must take a value between its lower bound v_{min} and upper bound v_{max}, respectively. *W* is the total weight of the truss, and the total number of elements is denoted by *n*. L_e, ρ_e, and A_e are respectively length, material density, and cross-sectional area of the *e*th element. The first frequency constraint (g_1) represents that some natural frequencies ω_j, should exceed the prescribed lower limits. The second frequency constraint (g_2) represents that other natural frequencies should be less than the prescribed upper limits. To handle optimization constraints, a penalty approach was utilized in this study by introducing the following pseudo-cost function:

$$f_{\text{cost}}\left(\left\{\mathrm{V}\right\}\right)=(1+\varepsilon_1 .\, \upsilon)^{\varepsilon_2} \times W\left(\left\{\mathrm{V}\right\}\right), \quad \upsilon=\sum_{j=1}^{2}\max\left[0,\; \mathrm{g}_j(\{\mathrm{V}\})\right] \tag{4}$$

where υ is the total constraint violation. Constants ε_1 and ε_2 must be selected considering the exploration and the exploitation rate of the search space. In this study, ε_1 was set equal to one while ε_2 was selected to decrease the total penalty. Thus, ε_2 increased from the value of 1.5 set in the first steps of the search process to the value of 3 set toward the end of the optimization process.

3. WATER EVAPORATION OPTIMIZATION ALGORITHM

Inspiring by the evaporation of a tiny amount of water molecules on the solid surface with different wettability which can be studied by molecular dynamics simulations, Kaveh and Bakhshpoori [13] developed a novel metaheuristic called Water Evaporation Optimization (WEO). The evaporation of water is very important in biological and environmental science. Based on the molecular dynamics simulations it is well-known that, as the surface changed from hydrophobicity to hydrophilicity, the evaporation speed does not show a monotonically decrease from intuition, but increases first, and then decreases after reaching a maximum value. When the surface wettability of the substrate is not high enough, the water molecules accumulate into the form of a sessile spherical cap. The predominant factor that affects the evaporation speed is the geometry shape of the water congregation. Meanwhile, when the surface wettability of the substrate is high enough, the water molecules spread to a monolayer and the geometric factor no longer affects much, and the energy barrier provided by the substrate instead geometry shape affects the evaporation speed.

WEO considers water molecules as algorithm individuals. A solid surface or substrate with variable wettability is reflected as the search space. Decreasing the surface wettability (substrate changed from hydrophilicity to hydrophobicity) reforms the water aggregation from a monolayer to a sessile droplet. Such behavior is consistent with how the layout of individuals changes to each other as the algorithm progresses. Decreasing the wettability of the surface can represent the decrease of the objective function for a minimizing optimization problem. The evaporation flux rate of the water molecules is considered as the most appropriate measure for updating the individuals whose pattern of change is in good agreement with the local and global search ability of the algorithm and can help WEO to have significantly well-converged behavior and simple algorithmic structure. In the following the WEO algorithm is organized in five steps and then its pseudocode and flowchart are presented [27].

Step 1: Initialization

Algorithm parameters are set in the first step. These parameters are the number of water molecules (*nWM*), and the maximum number of algorithm iterations (*maxNITs*). It should be noted that the minimum (MEP_{min}) and maximum (MEP_{max}) values of monolayer evaporation probability, and the minimum (DEP_{min}) and maximum (DEP_{max}) values of droplet evaporation probability can also be considered as the algorithm parameters. However, the evaporation probability parameters are determined efficiently for WEO based on the MD simulations results (MEP_{min}=0.03 and MEP_{max}=0.6; DEP_{min}=0.6 and DEP_{max}=1). WEO starts from *nWM* number of candidate solutions or water molecules randomly generated within the search space. These solutions construct the matrix of water molecules (*WM*). After evaluating the molecules, the corresponding objective function (*Fit*) and the penalized objective function (*PFit*) vectors are produced. In this study, *nWM* is considered as 10 and 20000 numbers of structural analysis are considered as the stopping criteria.

Step 2: Generating water evaporation matrix

Every water molecule follows the evaporation probability rules specified for each phase of the algorithm in the previous subsection.

Step 3: Generating random permutation-based step size matrix

A random permutation-based step size matrix is generated.

Step 4: Generating evaporated water molecules and updating the matrix of water molecules.

The evaporated set of water molecules *newWM* is generated by adding the product of step size matrix and evaporation probability matrix to the current set of molecules *WM* according to Eq. (13-11).

These new water molecules are evaluated based on the objective function. For molecule *i* (*i*=1, 2, …, *nWM*), if the newly generated molecule *i* (*i*=1, 2, …, *nWM*) is better than the old one it will replace it. The best water molecule (*bestWM*) is returned.

Step 5: Terminating condition

If the number of iterations of the algorithm (*NITs*) becomes larger than the maximum number of iterations (*maxNITs*), the algorithm terminates. Otherwise, go to Step 2.

The pseudo-code of WEO is given as follows:

Define the algorithm parameters: *nWM*, and *maxNFEs*.
Generate random initial water molecules (*WM*).
Evaluate the initial molecules, form their corresponding vectors of the objective function (*Fit*), and penalized objective function (*PFit*).
While *NFEs* ≤ *maxNFEs*
Update *NITs*.
if *NITs* ≤ *maxNITs*/2
Generate new water molecules based on the monolayer evaporation strategy.
Evaluate the newly generated water molecules and replace the current molecules with the evaporated ones if the newest ones are better.
Update *NFEs*.
else
Generate new water molecules based on the droplet evaporation strategy.
Evaluate the newly generated water molecules and replace the current molecules with the evaporated ones if the newest ones are better.
Update *NFEs*.
end
Determine and monitor the best water molecule (*bestWM*).
end While

4. NUMERICAL EXAMPLES

4.1 Planar 10-bar truss

Truss geometry including node and element numbering, a non-structural mass of 453.6 kg (1000 lb) is attached to all free nodes (1-4), and kinematic constraints are shown in Fig. 1.

The material is aluminum, with Young's modulus equal to 68.95 GPa and a specific mass of 2767.99 kg/m^3. These properties are according to a study by Miguel and Miguel [28]. The natural frequency constraints are $\omega_1 \geq 7$ Hz, $\omega_2 \geq 15$ Hz, and $\omega_3 \geq 20$ Hz. The allowable lower and upper bounds of the cross-sectional area (m^2) are 0.645×10^{-4} and 50×10^{-4}.

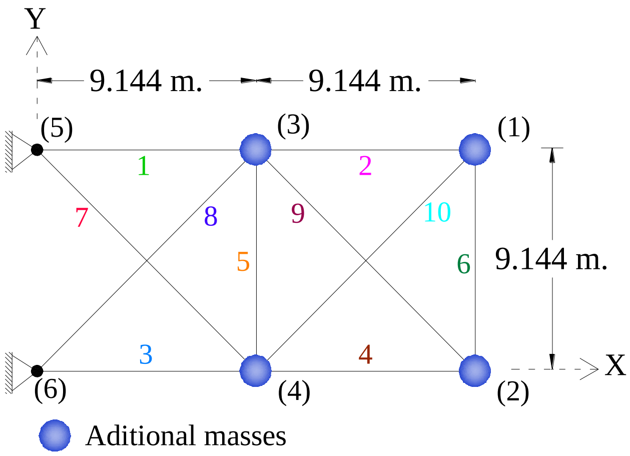


Figure 1. Schematic of the planar 10-bar truss structure [21].

Table 1 presents the best-optimized designs and the corresponding masses found by WEO and different methods: Firefly Algorithm (FA) [28], Harmoney Search (HS) [28], and Hybrid Particle Swarm and Swallow Swarm Optimization (HPSSO) algorithm [21]. Table 2 represents the corresponding natural frequencies. Table 3 presents the optimization results based on the WEO obtained for 30 independent runs carried out from different initial populations randomly generated and other methods. The number of independent runs is considered as 5 for FA and HS, and 30 for HPSSO. As it is clear WEO shows the best performance in terms of accuracy and also robustness.

Table 1. Optimization results (cm^2) were obtained by WEO and other metaheuristics in the 10-bar truss problem.

Member	FA [28]	HS [28]	HPSSO [21]	WEO this study
1	36.198	34.282	35.440	35.3931
2	14.030	15.653	14.807	14.8895
3	34.754	37.641	35.714	35.6111
4	14.900	16.058	14.975	14.8020
5	0.654	1.069	0.645	0.6450
6	4.672	4.740	4.620	4.6244
7	23.467	22.505	23.816	24.1682
8	25.508	24.603	24.253	23.9486
9	12.707	12.867	12.591	12.5054
10	12.351	12.099	12.526	12.7206
Mass (kg)	531.28	534.99	530.76	530.7294

Table 2. Optimum design of natural frequencies (HZ) for the 10 bar truss.

Frequency number	FA [28]	HS [28]	HPSSO [21]	WEO this study
1	7.0002	7.0028	7.000	7.0000
2	16.1640	16.7429	16.180	16.1709
3	20.0029	20.0548	20.001	20.0002
4	20.0221	20.3351	20.008	20.0027
5	28.5428	28.5232	28.545	28.5663
6	28.9220	29.2911	28.957	28.9944
7	48.3538	49.0342	48.556	48.5132
8	50.8004	51.7451	51.057	51.0229

Table 3. Comparison (kg) of robustness and reliability of WEO in the 10-bar truss problem.

Algorithm	Best	Average	Worst	SD
FA [28]	531.28	535.07	-	3.64
HS [28]	534.99	537.68	-	2.49
HPSSO [21]	530.7610	534.16	537.78	3.07
WEO (this study)	530.7294	532.4507	537.4512	2.18

4.2. Planar 37-bar truss

The second optimization problem is the simply supported planar 37-bar truss shown in Fig.2. A nonstructural mass of m=10 kg is attached at each of the free nodes on the lower chord. The steel material has a modulus of elasticity of 210 Gpa and a density of 37800 kg/m^3. The truss is optimized on shape and size for its mass minimization with multiple frequency constraints. Nodal coordinates in the upper chord and cross-sectional areas of members are considered as design variables. All members on the lower chord have fixed cross-sectional areas of 4×10^{-3} m^2 and the others have initial cross-sectional areas of 1×10^{-4} m^2 (also as the lower bound). In the optimization process, nodes on the upper chord can be shifted vertically. In addition, nodal coordinates and member areas are linked to maintaining structural symmetry. Therefore, only five shape variables and fourteen sizing variables will be redesigned for optimization. The natural frequency constraints are $\omega_1 \geq 20$ Hz, $\omega_2 \geq 40$ Hz, and $\omega_3 \geq 60$ Hz.

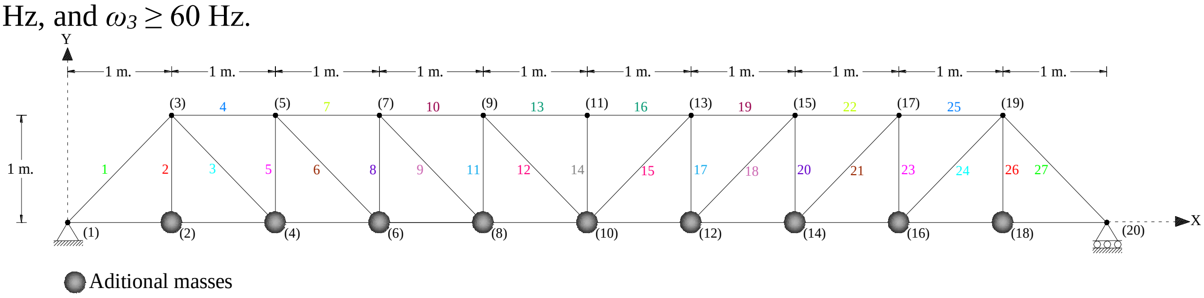


Figure 2. Schematic of the 37-bar truss structure [21].

Table 4 presents the best-optimized designs and the corresponding masses found by HPSSO and different methods (FA, HS, CSS, standard PSO, and DPSO), and Table 5 represents the corresponding natural frequencies. Table 6 presents the statistical results based on the HPSSO obtained for 30 independent runs carried out from different initial populations randomly generated and other methods. The number of independent runs is considered as 5 for FA and HS, 10 for CSS, and 30 for both standard and democratic PSO. Comparing the results reveal that WEO has better performance in terms of accuracy than other algorithms except for HPSSO which is a hybridized version of the PSO algorithm so the difference is too small. Robustness comparison shows that WEO is the most successful algorithm.

Table 4. Comparison of optimization results (Y coordinates: m; and areas: cm^2) obtained by WEO in the 37-bar truss problem.

Member	FA [28]	HS [28]	CSS [29]	HPSSO [21]	WEO this study
Y3, Y19	0.9392	0.8415	0.8726	1.00000	1.0295
Y5, Y17	1.3270	1.2409	1.2129	1.357692	1.3998
Y7, Y15	1.5063	1.4464	1.3826	1.531195	1.5888
Y9, Y13	1.6086	1.5334	1.4706	1.666696	1.7255
Y11	1.6679	1.5971	1.5683	1.734591	1.8003
A1, A27	2.9838	3.2031	2.9082	2.911875	2.7447
A2, A26	1.1098	1.1107	1.0212	1.00000	1.0000
A3, A24	1.0091	1.1871	1.0363	1.00000	1.0104
A4, A25	2.5955	3.3281	3.9147	2.539312	2.5579
A5, A23	1.2610	1.4057	1.0025	1.268065	1.1710
A6, A21	1.1975	1.0883	1.2167	1.135538	1.2454
A7, A22	2.4264	2.1881	2.7146	2.546305	2.4438
A8, A20	1.3588	1.2223	1.2663	1.392601	1.4537
A9, A18	1.4771	1.7033	1.8006	1.432117	1.4945
A10, A19	2.5648	3.1885	4.0274	2.492398	2.1299
A11, A17	1.1295	1.0100	1.3364	1.174892	1.1738
A12, A15	1.3199	1.4074	1.0548	1.352078	1.4040
A13, A16	2.9217	2.8499	2.8116	2.57735	2.4162
A14	1.0004	1.0269	1.1702	1.00000	1.0030
Mass (kg)	360.05	361.50	362.84	359.975	360.0968

Table 5. Optimum design of natural frequencies (HZ) for the 37-bar truss.

Frequency number	FA [28]	HS [28]	CSS [29]	HPSSO [21]	WEO this study
1	20.0024	20.0037	20.0000	20.0092	20.0039
2	40.0019	40.0050	40.0693	40.0222	40.0173
3	60.0043	60.0082	60.6982	60.0186	60.0024
4	77.2153	77.9753	75.7339	76.2377	76.1241
5	96.9900	99.2564	97.6137	95.5098	95.5010

Table 6. Comparison (kg) of robustness and reliability of WEO and other metaheuristic methods in the 37-bar truss problem.

Algorithm	Best	Average	Worst	SD
FA [28]	360.05	360.37	-	0.26
HS [28]	361.50	362.04	-	0.52
CSS [29]	362.84	366.77	-	3.742
HPSSO [21]	359.975	364.1593	398.4291	7.864
WEO (this study)	360.0968	361.6003	368.2501	1.6607

4.3. The 52-bar dome shaped truss

The third numerical test case is simultaneous layout and size optimization of a 52-bar domelike truss. The initial layout of the structure is depicted in Fig. 3.

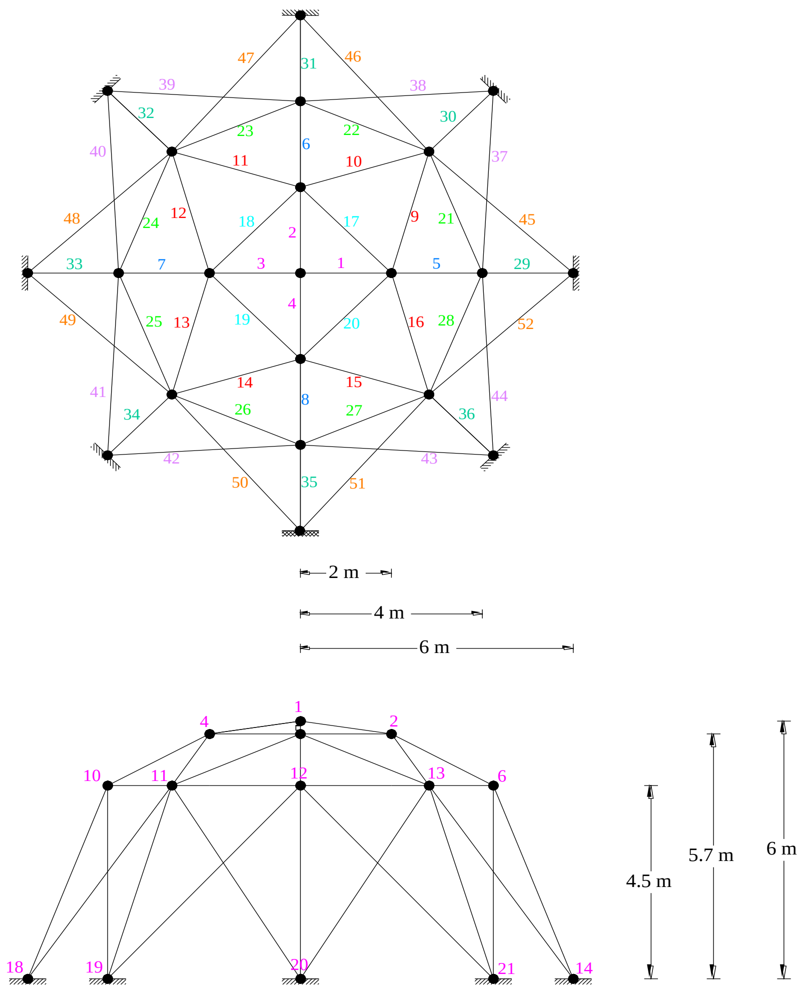


Figure 3. Schematic of the dome shaped 52-bar truss structure [21].

This test case is described in detail in Ref. [21]. The optimized designs found by the different algorithms are compared in Table 7 that shows also the corresponding structural weights, and Table 8 represents the corresponding natural frequencies. Statistical results of independent optimization runs are presented in Table 9. Performance comparison shows that WEO performs like the previous example in this test case.

Table 7. Comparison of optimization results (Y coordinates: m; and areas: cm^2) obtained by WEO and in the 52-bar truss problem.

Member	FA [28]	HS [28]	CSS [29]	HPSSO [21]	WEO this study
Z1	6.4332	4.7374	5.2716	5.9086	5.9257
X2	2.2208	1.5643	1.5909	2.2106	2.2291
Z2	3.9202	3.7413	3.7093	3.7742	3.7561
X6	4.0296	3.4882	3.5595	3.9859	3.9777
Z6	2.5200	2.6274	2.5757	2.5007	2.5027
A1	1.0050	1.0085	1.0464	1.0000	1.0078
A2	1.3823	1.4999	1.7295	1.1800	1.1822
A3	1.2295	1.3948	1.6507	1.2686	1.2811
A4	1.2662	1.3462	1.5059	1.4268	1.4584
A5	1.4478	1.6776	1.7210	1.4380	1.4275
A6	1.0000	1.3704	1.0020	1.0000	1.0001
A7	1.5728	1.4137	1.7415	1.5553	1.5452
A8	1.4153	1.9378	1.2555	1.4083	1.4106
Mass (kg)	197.53	214.94	205.237	195.1085	195.2988

Table 8. Optimum design of natural frequencies (HZ) for the 52-bar truss.

Frequency number	FA [12]	HS [12]	CSS [11]	HPSSO [21]	WEO this study
1	11.3119	12.2222	9.246	11.4099	11.4746
2	28.6529	28.6577	28.648	28.6483	28.6498
3	28.6529	28.6577	28.699	28.6490	28.6572
4	28.8030	28.6618	28.735	28.7166	28.7220
5	28.8030	30.0997	29.223	29.1050	29.0268

Table 9. Comparison (kg) of robustness and reliability of WEO in the 52-bar truss problem.

Algorithm	Best	Average	Worst	SD
FA [28]	197.53	212.80	-	17.98
HS [28]	214.94	229.88	-	12.44
CSS [29]	205.237	213.101	-	7.391
HPSSO [21]	195.1085	214.0870	270.0908	19.8910
WEO (this study)	195.2988	197.9313	204.6654	2.9400

4.4. The 72-bar space truss

The spatial 72-bar truss is considered the last test case. IT is schematized in Fig. 4 [21]. In the four nodes on the top of the structure (nodes 1–4), it is attached a non-structural mass of 2270 kg. The design variables are the member cross-sectional areas, treated as continuous design variables, which are linked into 16 groups to maintain the structural symmetry. Member linking detail is available in Table 14 in Ref. [28]. The material is aluminum, with Young's modulus equal to 68.95 GPa and a specific mass of 2770 kg/m^3. These properties are according to the study by Miguel and Miguel [28] using FA and HS. The natural frequency constraints are ω_1 = 4 Hz and $\omega_3 \geq 6$ Hz. The allowable minimum area of the cross-sectional is 0.645×10^{-4} m^2.

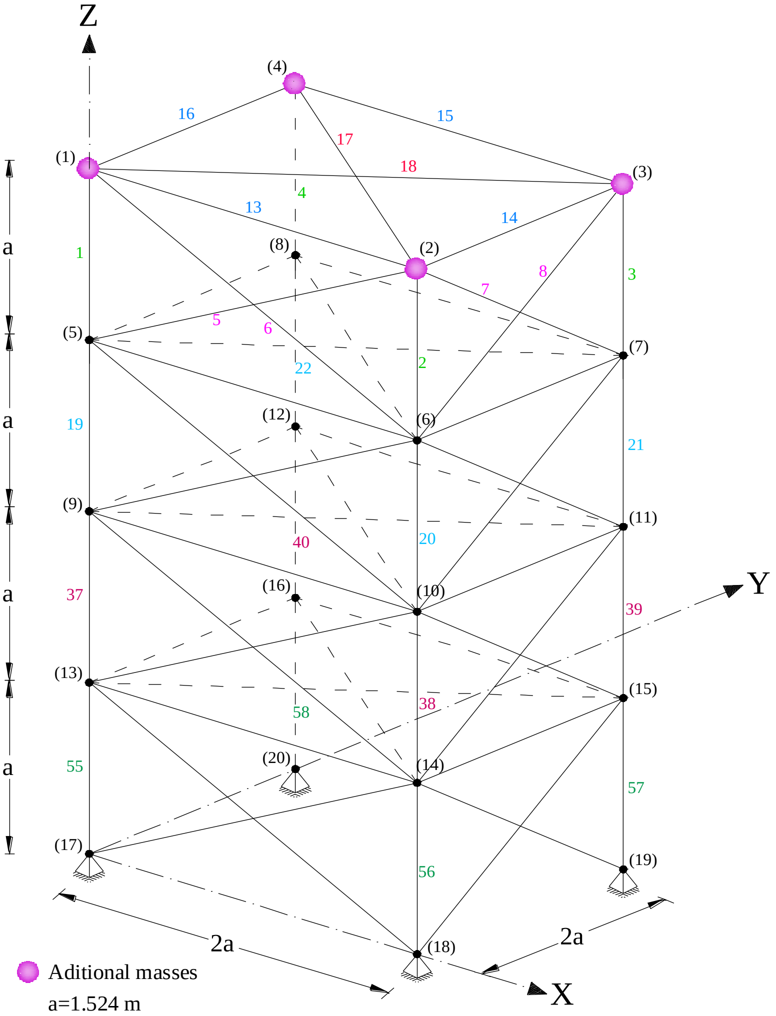


Figure 4. Schematic of the spatial 72-bar truss structure.

Table 10 tabulates the best-optimized designs and the corresponding masses found by different methods and WEO. It should be noted that this test case is considered with different details in the literature. Therefore this table contains two columns for WEO. The first column compares WEO with FA, HS, and HPSSO and the second one compares WEO with CSS, and HPSSO. Table 11 represents the corresponding natural frequencies. Table 12 presents the optimization results based on the WEO obtained for 30 independent runs carried out from different initial populations randomly generated and other methods. The number of independent runs is considered as 5 for FA, HS, and TLBO, 10 for CSS, and 30 for HPSSO. It should be noted that this test case is considered with different details in the literature. Therefore this table contains two rows for WEO. The first row compares WEO with FA, HS, and HPSSO and the second one compares WEO with CSS and HPSSO. As it is clear WEO is again the best algorithm in terms of robustness and also shows a comparative performance in terms of accuracy.

Table 10. Optimization results (cm^2) were obtained by WEO in the 72-bar truss problem.

Member	FA [28]	HS [28]	HPSSO [21]	CSS [29]	HPSSO [21]	WEO this study
1	3.3411	3.6803	3.5329	2.528	3.4041	3.7448
2	7.7587	7.6808	8.0157	8.704	7.5881	8.1431
3	0.6450	0.6450	0.6450	0.645	0.6451	0.6457
4	0.6450	0.6450	0.6450	0.645	0.6451	0.6468
5	9.0202	9.4955	8.0510	8.283	8.2960	7.7538
6	8.2567	8.2870	7.9363	7.888	7.7144	7.9413
7	0.6450	0.6450	0.6450	0.645	0.6450	0.6493
8	0.6450	0.6461	0.6450	0.645	0.6450	0.6475
9	12.0450	11.4510	12.6954	14.666	12.4260	13.1020
10	8.0401	7.8990	8.0952	6.793	8.2415	7.9536
11	0.6450	0.6473	0.6450	0.645	0.6450	0.6450
12	0.6450	0.6450	0.6470	0.645	0.6455	0.6452
13	17.3800	17.4060	17.3953	16.464	17.0557	17.1117
14	8.0561	8.2736	8.0887	8.809	8.2833	8.1000
15	0.6450	0.6450	0.6455	0.645	0.6450	0.6459
16	0.6450	0.6450	0.6450	0.645	0.6450	0.6450
Mass (kg)	327.691	328.334	327.6923	328.814	324.7630	327.8048

Table 11. Optimum design of natural frequencies (HZ) for the 72 bar truss.

Frequency number	FA [28]	HS [28]	CSS [29]	HPSSO [21]	WEO This study
1	4.0000	4.0000	4.000	4.0000	4.0000
2	4.0000	4.0000	4.000	4.0001	4.0001
3	6.0000	6.0000	6.006	6.0004	6.0001
4	6.2468	6.2723	6.21	6.2459	6.2612
5	9.0380	9.0749	8.684	9.0801	9.1223

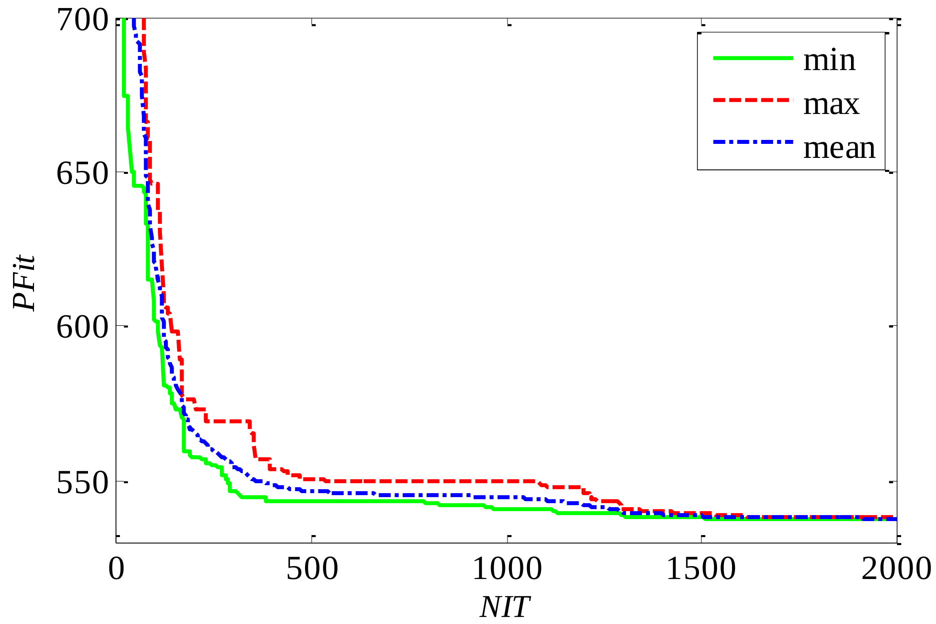
*According to the details used by FA and HS [28]

Table 12. Comparison (kg) of robustness and reliability of WEO in the 72-bar truss problem.

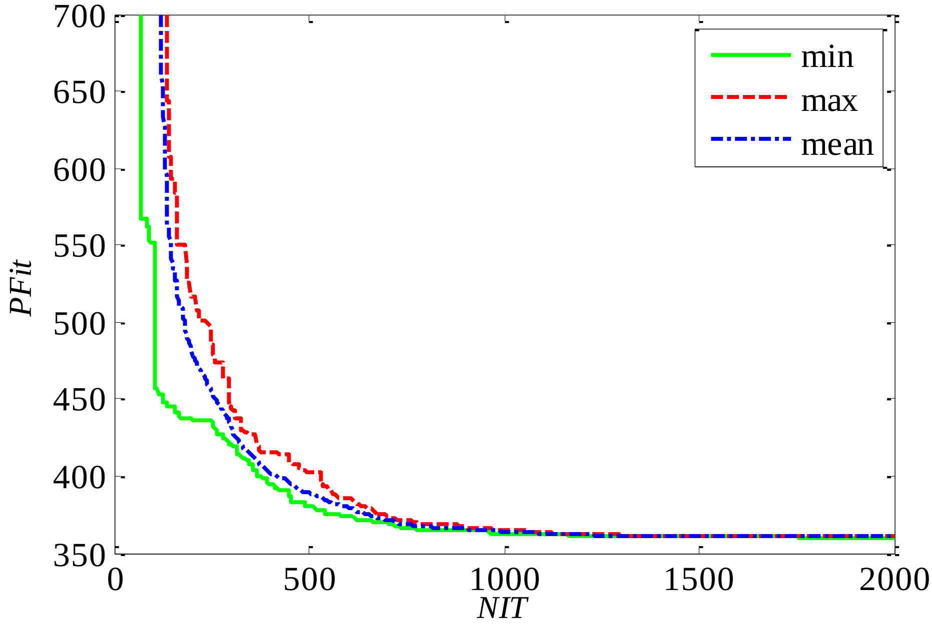
Algorithm	Best	Average	Worst	SD
FA [28]	327.691	329.89	-	2.59
HS [28]	328.334	332.64	-	2.39
CSS [29]	328.814	337.70	-	5.42
HPSSO [21]	324.7630	330.98	401.68	15.57
WEO (this study)	327.8048	328.1885	328.8435	0.2730

5. CONVERGANCE BEHAVIOUR OF WEO

To make a deep evaluation of the performance of the WEO this section monitor and discuss its convergance behaviour. To do so first the convergance history of the WEO for a single run of trusses is depicted in Fig. 5. Convergence curves compare the history of the best, worst and average values of all particles. As it is clear WEO shows a good convergence speed for all test trusses and also in all the test cases, WEO can preserve the diversification until the last iterations of the optimization procedure.

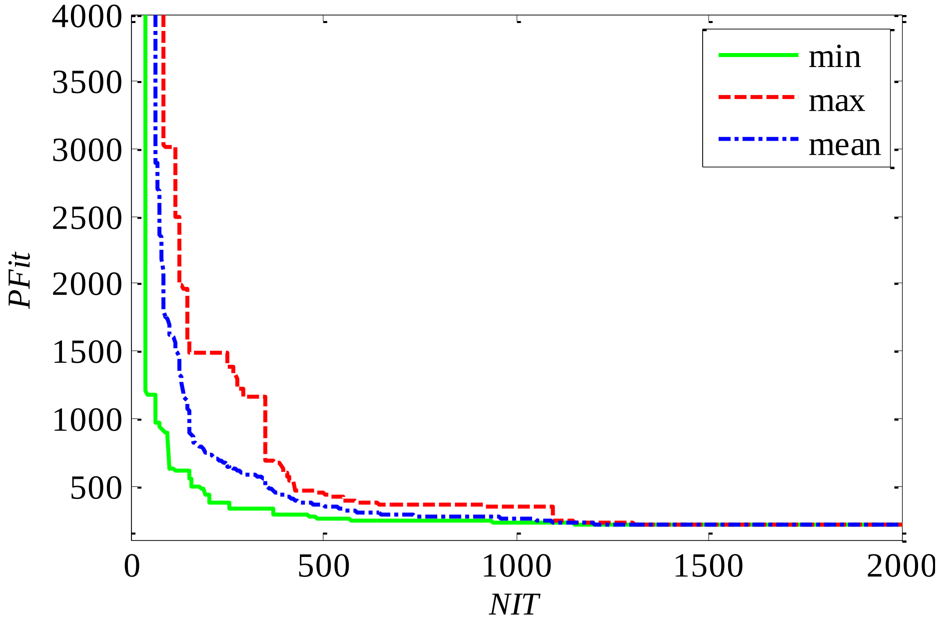


(a)



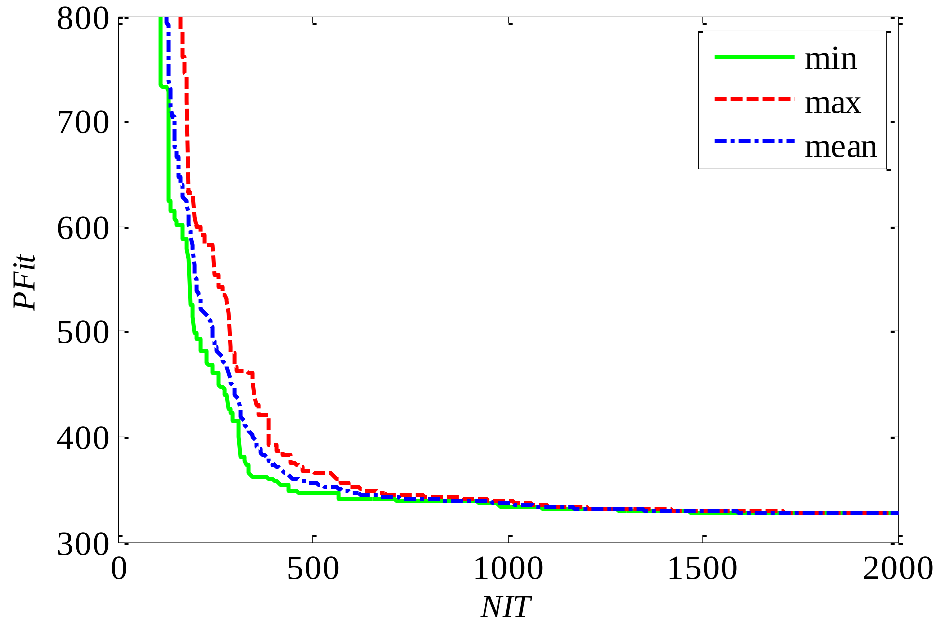
PFit
700
650
600
550
500
450
400
350
0
500
1000
1500
2000
NIT
min
max
mean

(b)



PFit
4000
3500
3000
2500
2000
1500
1000
500
0
500
1000
1500
2000
NIT
min
max
mean

(c)



(d)

Figure 5. Comparison of the convergence curves recorded for the best and worst particles and the average of all particles for the test problems: (a) 10-bar, (b) 37-bar, (c) 52-bar, and (d) 72-bar.

Any algorithm with the ability to make a good balance between exploration (diversification) and exploitation (intensification) is a favorite choice for optimization practitioners. This ability generally can guarantee robustness and adaptability. MHs start with a high diversity and tend toward high intensification as the optimization process progresses. In a nutshell, it should be mentioned that diversification should be made comprehensively to explore the search space and intensification should be made by clever exploitation at the right time and right place to ensure the convergence. Diversification should precede intensification because the algorithm needs a superior knowledge of the search space to make a clever intensification, and this can be accomplished at the expense of the convergence rate. Therefore diversity is one factor that can affect the more general concept of exploration and exploitation and any algorithm should preserve an efficient diversification till its end. To assess the performance of the MHs in the aspect of diversity, Kaveh and Zolghadr [30] defined a diversity index (DI) which reflects the ratio of the portion of the search space covered by the population to the entire search space at each iteration. Considering the number of the population of the metaheuristic and the number of design variables equal to NP and nV, respectively, it is formulated mathematically as:

$$DI = \frac{1}{NP}\sum_{j=1}^{NP}\sqrt{\sum_{i=1}^{nV}\left(\frac{GB(i)-X_j(i)}{x_{i,\max}-x_{i,\min}}\right)^2} \quad (4)$$

where $X_j(i)$ is the value of the ith variable of the jth individual; $GB(i)$ is the ith variable of the best candidate solution found by the algorithm till the iteration; and $x_{i,min}$ and $x_{i,max}$ are the minimum and maximum values of the ith design variable, respectively. The diversity index represents the distribution of the solution candidates around the best solution. Fig. 6 shows the variation of the DI for a single run of the WEO for all trusses concerning the iteration number. A desirable trend of variation of the diversity index is obtained by WEO. Up down step-like movements of the DI convergence history in the early stages of the optimization process show how WEO covers numerous promising points of the search space. High values of diversity are provided in the early stages of the optimization process. As the optimization process continues, the water molecules focus on more promising regions of the search space to perform a local search, and diversity index values gradually decrease.

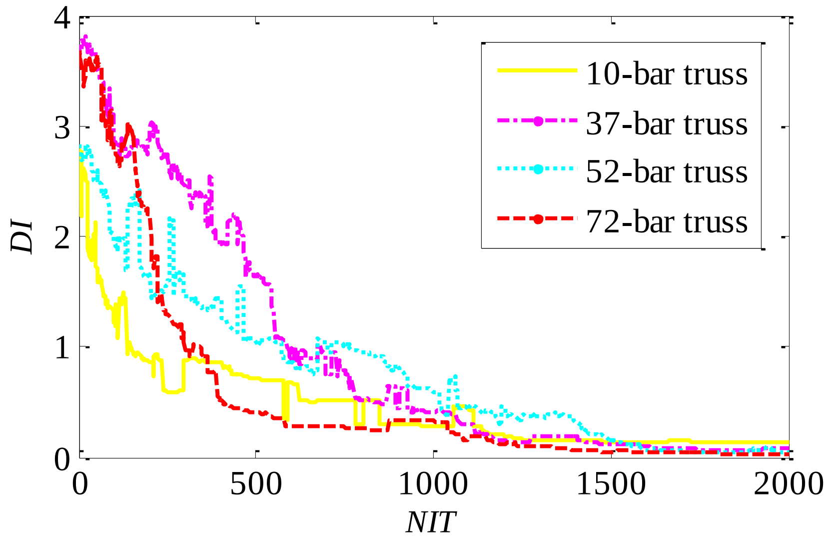


Figure 6. Values of diversity index recorded in the optimization history for a single run of different test problems

6. CONCLUSION

Water evaporation optimization algorithm is applied to address the truss layout and size optimization considering dynamic constraints. This type of structural optimization problem is well known to be highly nonlinear and non-convex. Four benchmark numerical trusses are considered to evaluate the performance of WEO in comparison with other well-known metaheuristics. Simulation results reveal that WEO performs comparatively to the most well-known algorithms, high performance in comparison to those of different optimization techniques, and high performance in comparison to all algorithms in terms of robustness. Such a performance is due to the good balance between global and local exploration abilities of WEO which can make it a suitable algorithm with robust efficiency for other complicated constrained engineering optimization problems.

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