BIG BANG – BIG CRUNCH ALGORITHM FOR LEAST-COST DESIGN OF WATER DISTRIBUTION SYSTEMS

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ABSTRACT

The Big Bang-Big Crunch (BB–BC) method is a relatively new meta-heuristic algorithm which inspired by one of the theories of the evolution of universe. In the BB–BC optimization algorithm, firstly random points are produced in the Big Bang phase then these points are shrunk to a single representative point via a center of mass or minimal cost approach in the Big Crunch phase. In this paper, the BB–BC algorithm is presented for optimal cost design of water distribution systems and employed to optimize different types of hydraulic networks with discrete variables. The results demonstrate the efficiency of the proposed method compared to other algorithms.

1. INTRODUCTION

During the last decades, several new nature-inspired optimization algorithms based on heuristic search techniques have been proposed and widely used to solve complex combinatorial optimization problems encountered in a variety of real-life applications. These meta-heuristics have two things in common: (1) they are stochastic and (2) they are inspired

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from nature. Also, they do not require for a given function to be derivable and an explicit relationship between the objective function and constraints is not needed.

On the other hand, obtaining the least cost design of water distribution network is a combinatorial problem, that is, a set of solutions must be selected from a discrete set of feasible solutions while the functions represent the hydraulic behavior of the network are nonlinear [1]. The solution process involves simultaneous consideration of continuity equation, energy conservation, and head-loss function that makes the analytical solution of the problem becomes complicated. From a mathematical point of view, significant difficulties are involved due to the discrete nature of the pipe diameters and the nonlinearity of the head-loss relationship. These lead to a large-scale, mixed integer, and nonlinear problem, corresponding to the NP-hard class. Therefore, in the last few years, the popularity of meta-heuristics in optimization of water distribution systems has steadily increased; the reason for this is that sometimes, on large-scale real-world problems, meta-heuristics are the only viable tools for optimization.

Big Bang–Big Crunch (BB–BC) algorithm [2] is a reative newly proposed meta-heuristic optimization method inspired from the theories of the universe evolution. Due to the simple algorithmic outline of the algorithm and its efficiency in solving optimization problems, it has become one of the popular meta-heuristics of the recent years [3]. This algorithm has been applied to many areas including fuzzy model inversion [4], genetic programming classifier design [5], optimum design of structures [6-9], and optimal solution of large scale reservoir operation problems [10]. For the first time, this paper applies a discrete version of BB–BC for optimal cost design of water distribution systems. In practical optimization problems, industrial pipe diameters are used which have discrete values and as a result a discrete solution is more suitable than continuous one for this kind of the optimization problems.

In this study, the BB-BC is coupled with the widely used water distribution network software, EPANET 2, and applied to the water distribution network design, in which the BB-BC optimization model is the outer driver model and simulation is the inner model. EPANET Programmer's Toolkit was provided by the United States Environmental Protection Agency (USEPA) that is a dynamic link library (DLL) of functions which allows developers to customize EPANET's computational engine for the user's specific needs. Thus, a computer programming codes is written for the BB-BC in MATLAB, and EPANET 2 is linked via the EPANET Toolkit.

2. BB-BC OPTIMIZATION

The Big Bang-Big Crunch (BB–BC) optimization method developed by Erol and Eksin [2] consists of two main steps: The first step is the Big Bang phase where candidate solutions are randomly distributed over the search space, and the next step is the Big Crunch phase where a contraction procedure calculates a center of mass for the population. Here, the term mass refers to the inverse of the fitness function value. The point representing the center of mass, denoted by $\vec{x}^c$, is calculated as

$$\vec{x}^c = \frac{\sum_{i=1}^{N} \frac{1}{f^i} \vec{x}^i}{\sum_{i=1}^{N} \frac{1}{f^i}} \qquad (1)$$

where $\vec{x}^i$ is a point within an n-dimensional search space generated, f^i is a fitness function value of this point, N is the population size in Big Bang phase. After the Big Crunch phase, the algorithm creates the new solutions to be used as the Big Bang of the next iteration step by using the center of mass. This can be accomplished by spreading new off-springs around the center of mass using a normal distribution operation in every direction, where the standard deviation of this normal distribution function decreases as the number of iterations increases:

$$\vec{x}_{new}^i = \vec{x}^c + \frac{ra(x_{max} - x_{min})}{k+1}, \quad i=1,2,...,N \qquad (2)$$

where x_{new}^i is the position of the new candidate solution i, r is a random number from a standard normal distribution which is different for each candidate, α is a parameter for limiting the size of the search space, and k is the number of iteration.

For discrete variables or when index numbers are related to tabular discrete values, the continuous values x_{new}^i are compared to the entire x^i and the nearest value is selected as x_{new}^i. Since normally distributed numbers can exceed ± 1, it is necessary to limit the candidate positions to the prescribed search space boundaries. A consequence of this contraction is the accumulation of candidate solutions at the search space boundaries.

The pseudo-code of the BB–BC algorithm can be summarized as follows [3]:

Step 1: Form the initial population by spreading randomly solution candidates over all search space (first Big Bang) in a uniform manner. This step has to be applied once.

Step 2: Calculate the fitness value of every individual point and assign this value as its mass (if a minimization is to be carried out, form the "mass value" either by inversing the fitness/cost value or by subtracting it from a constant number chosen bigger than the maximum possible value).

Step 3: Calculate the "center of mass" by taking the weighted average using the coordinates and the mass values of every single individual (Big Crunch phase) or choose the fittest individual among all as their center of mass.

Step 4: Generate new solution candidates by using normal distribution (Big Bang phase).

Step 5: Keep the fittest individual found so far in a separate place or as a member of the population (elitism) and go to step 2 until a stopping criterion is accomplished.

3. STATEMENT OF THE OPTIMIZATION PROBLEM

Generally the optimization problem of water distribution network can be defined as: how to supply an adequate water quantity in order to cover the needed demand for each node through a highly interconnected system of pipes, and through using network elements such as pumps, reservoirs and tanks. The mathematical statement of the optimal design problem can be

written as:

Minimize:

$$f_{\cos t} = \sum_{i=1}^{N_p} f(D_i, L_i) \tag{3}$$

Subject to:

$$\begin{cases} \sum Q_{in} - \sum Q_{out} = \sum Q_e \\ \sum h_f - \sum E_p = 0 \\ H_j \geq H_j^{\min} \end{cases} \tag{4}$$

where $f(D_i, L_i)$ is the cost of pipe i with diameter D_i and length L_i, and N_p is the number of pipes in the network. In continuity constraint Q_{in} is the flow rate to the node, Q_{out} is the flow rate out of the node, and Q_e is the external inflow rate at the node. In energy constraint h_f is the head loss computed by the Hazen-Williams or Darcy-Weisbach formulae and E_p is the energy added to the water by a pump. Also H_j is the pressure head and H_j^{min} is the minimum required pressure head at node j in which $j=1, 2,...,N_n$. N_n is the number of nodes in the network.

Different forms for the head loss formula have been developed for practical pipe flow calculations. In this study, the head loss (h_f) in the pipe is expressed by the Hazen-Williams formula, as:

$$h_f = w \frac{L}{C^a D^b} Q^a \tag{5}$$

where $\omega=10.6668$, $\alpha=1.85$, $\beta=4.87$, Q_i is the pipe flow (m³/s), C_i is the is the Hazen-Williams roughness coefficient which ranges from 150 for smooth-walled pipes to as low as 80 for old, corroded cast iron pipes, D_i is pipe diameter (m), and L_i is pipe length (m).

In order to handle the constraints, a penalty approach is utilized. If the constraints are between the allowable limits, the penalty is zero; otherwise the amount of penalty is obtained by dividing the violation of allowable limit to the limit itself. After analyzing a model, the pressure of each node is obtained then these values are compared to the allowable limits to calculate the penalty functions as:

$$\begin{cases} H_j^{\min} \leq H_j & \Rightarrow \Delta^{(j)} = 0 \\ H_j^{\min} > H_j & \Rightarrow \Delta^{(j)} = \dfrac{H_j^{\min} - H_j}{H_j^{\min}} \end{cases} \quad j=1,2,....,N_n \tag{6}$$

In this method, the aim of the optimization is redefined by introducing the cost function as:

$$F_{\text{cost}} = (1 + e_1 \cdot \sum \Delta)^{e_2} \times f_{\cos t} \tag{7}$$

The penalty function method has certain drawbacks, for example penalty parameters are

problem dependent and needs proper parameter tuning to converge to the feasible domain. Here, the constants e_1 and e_2 are selected considering the exploration and the exploitation rate of the search space. ε_1 is set to unity and ε_2 is selected in a way that it decreases the penalties and reduces the cost. Thus, in the first steps of the search process, ε_2 is set to 1.05 and ultimately increased to 1.2.

4. DESIGN EXAMPLES

4.1. Hanoi water distribution network

The first problem is proposed by Fujiwara and Khang [11]. This network consists of 32 nodes, 34 pipes and 3 loops. The network has no pumping station as it is fed by gravity from a reservoir with a 100 m fixed head. For this example, the system data are presented in Table 1. Hanoi network (Figure 1) requires the optimal design of 34 pipes, allowing a minimum hydraulic head of 30 m for all its 32 nodes, by means of 6 available diameters. The total solution space is then equal to $6^{34} = 2.87 \times 10^{26}$. The cost of commercially available pipe sizes {12, 16, 20, 24, 30, 40; in inches} is {45.73, 70.40, 98.38, 129.30, 180.80, 278.30 in dollar/meter}.
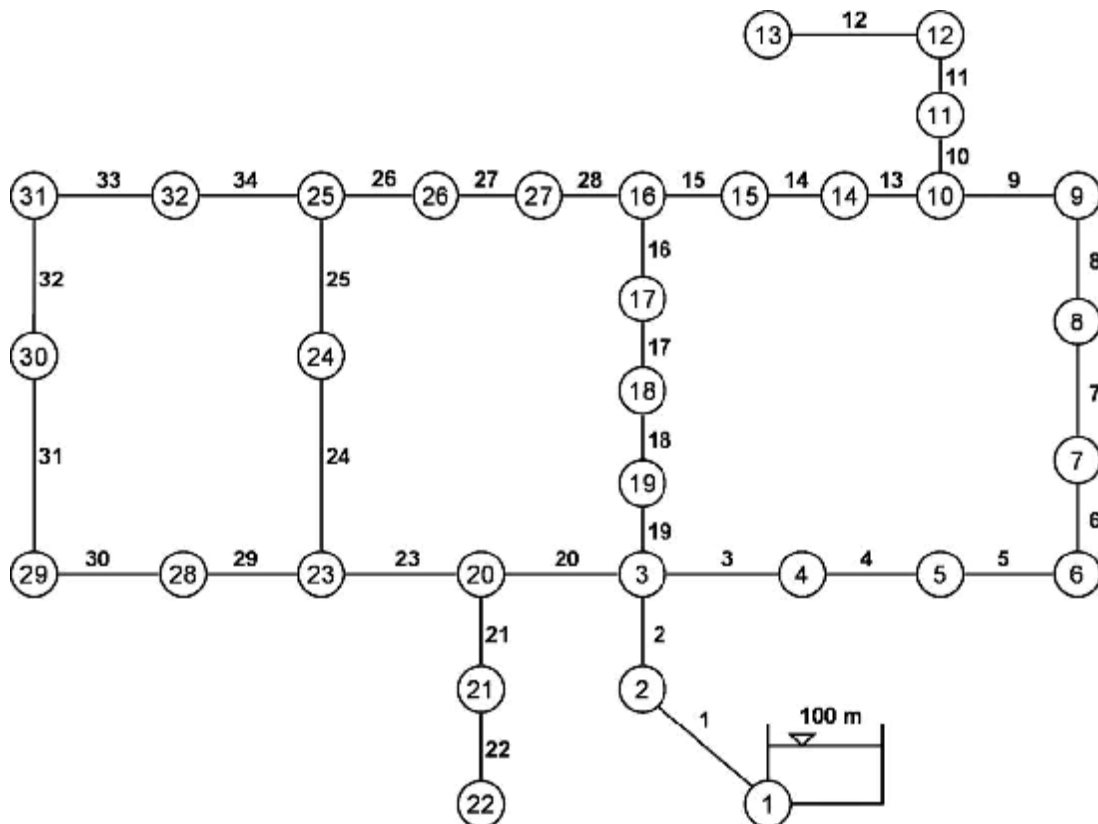

Figure 1. Network layout for the Hanoi problem

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Table 1. Hanoi network data

Node number	Demand (m³/h)	Pipeline	Length (m)	BB–BC optimal diameter (in)
01	–	01	100	40
02	890	02	1350	40
03	850	03	900	40
04	130	04	1150	40
05	725	05	1450	40
06	1005	06	450	40
07	1350	07	850	40
08	550	08	850	30
09	525	09	800	30
10	525	10	950	30
11	500	11	1200	24
12	560	12	3500	24
13	940	13	800	12
14	615	14	500	12
15	280	15	550	16
16	310	16	2730	30
17	865	17	1750	30
18	1345	18	800	30
19	60	19	400	30
20	1275	20	2200	40
21	930	21	1500	20
22	485	22	500	12
23	1045	23	2650	30
24	820	24	1230	24
25	170	25	1300	20
26	900	26	850	16
27	370	27	300	24
28	290	28	750	24
29	360	29	500	16
30	360	30	2000	16
31	105	31	1600	12
32	805	32	150	16
		33	860	20
		34	950	24

Table 2 reports the best results and the required number of analyses for convergence in the present algorithm and some of other heuristic methods. In this example, a population of 50 individuals is used and the BB–BC found the best feasible solution of 6.224×10^6 \$ after 520 iteration (26,000 analyses). The best cost of the BLIP (binary linear integer programming), MSATS (mixed simulated annealing and tabu search), SSSA (scatter search using SA as local searcher), [12] and SCE (shuffled complex evolution) [13] is 6.363, 6.352, 6.273, and 6.220 million dollars, respectively. In addition the BLIP, MSATS, and SSSE found the best feasible solution after 26,457 function evaluations.

Table 2. Performance comparison for the Hanoi network

Method*	Cost (10^6\$)	No. of analysis
BB–BC	6.224	26,000
BLIP [12]	6.363	26,457
MSATS [12]	6.352	26,457
SSSA [12]	6.273	26,457
SCE [13]	6.220	25,402

* **For all methods** $\omega = 10.6668$

4.2. Double hanoi network

The second design example is a double Hanoi network. Because this network is derived from the basic Hanoi network, its optimal cost is known. All the parameters for the reservoir, nodes and lines in the double Hanoi water distribution network are the same as in the original Hanoi network on both mirrored parts except for the first pipe (from the reservoir to node 2), which is shortened from the original 100 to 28.9 m. This change was made for the sake of obtaining the same head in node 2 (with a diameter of 40 in, which will certainly be proposed here by any optimization method) as in the original Hanoi network. Network layout for this problem is shown in Figure 2. The *reference* optimal solution (global) could be evaluated as follows [14]:

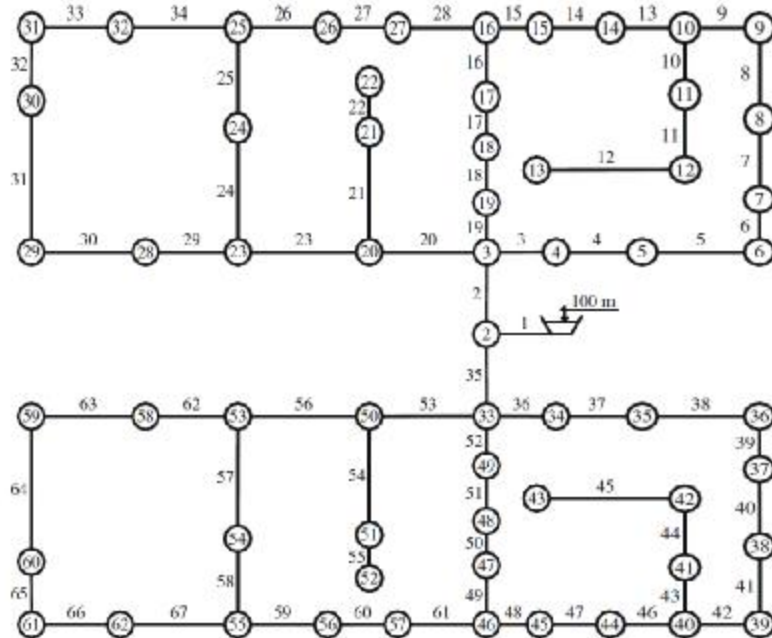$$C_{DH} = 2C_H - 2L_1C_1 + 28.9C_1 \tag{8}$$

Figure 2. Double Hanoi network

In which C_{DH} is the optimal cost of the double Hanoi network; C_H is the reference optimal cost of the Hanoi network (6.081×10^6 \$); L_l is the length of the first pipe on the original network (100 m); and C_1 is the unit price of the diameter 40 in (278.28 \$).

For our solution described in the previous example (6.224×10^6 \$), according to Eq. 8 the global optimum solution of the double Hanoi network should be 12.400×10^6 \$. The best results obtained with the BB–BC, GA, OptiDesigner, and the HS [14] are summarized in Table 3. The reference optimal cost of the Hanoi network for the GA and HS is 6.081×10^6 \$ and it is 6.115×10^6 \$ for OptiDesigner. The BB-BC found the best feasible solution of 12.648×10^6 \$ after 36,000 analysis while the best cost for the HS, GA, and OptiDesigner are 12.405, 12.601, and 12.795 million dollars, respectively. Also, deviation from global optimum for the BB–BC algorithm is 2.00%, and it is 2.39%, 4.01%, and 5.62% for the HS, GA, and OptiDesigner, respectively. This result demonstrates that the BB–BC algorithm is better in term of closeness to the global minimum. The Convergence history for double Hanoi network using the BB–BC algorithm and the hydraulic head for each nude are shown in Figures 3 and 4, respectively. As shown in Figure 4, the minimum value for pressure head is equal to 30.0448 m (at node 59).

Table 3. Performance comparison for the double Hanoi network

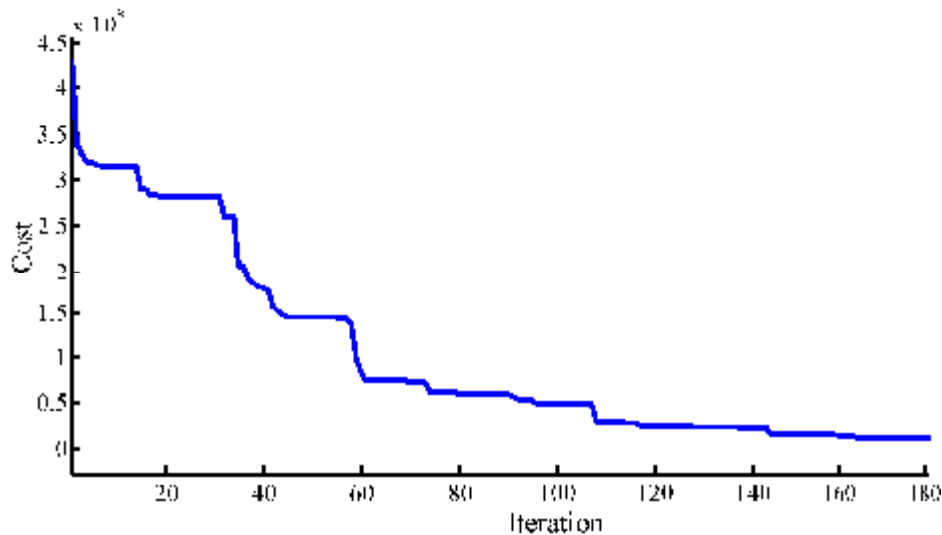| Method	Double Hanoi network	Deviation from reference global optimum (%)
BB–BC	12,647,789	2.00
HS [14]	12,404,680	2.39
GA [14]	12,600,624	4.01
OptiDesigner [14]	12,795,541	5.62

Figure 3. The convergence for the double Hanoi network using BB–BC
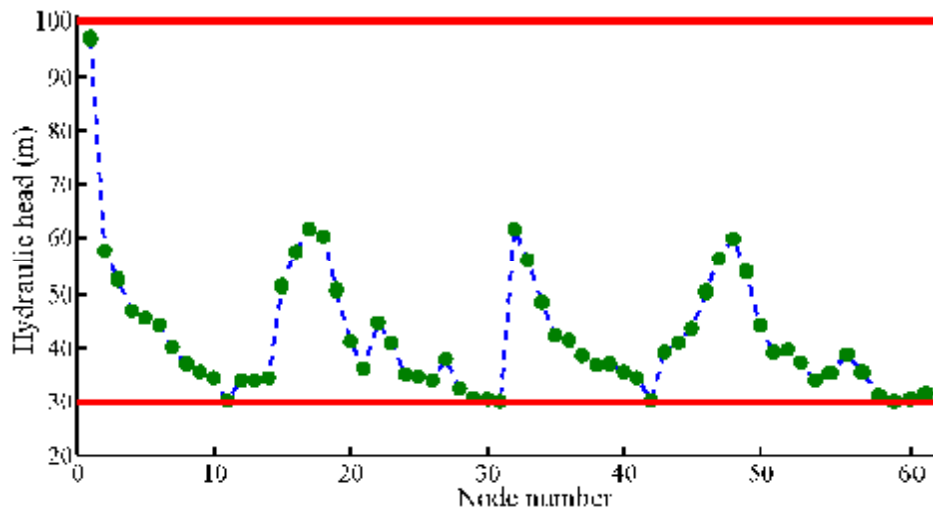
Figure 4. Comparison of the allowable and existing hydraulic head for the nodes of the double Hanoi network using BB–BC

5. CONCLUSION

This paper is the first attempt to apply the BB–BC for solving the least-cost design problem of water distribution networks. The BB–BC algorithm consists of two main steps: in the Big Bang phase; energy dissipation produces a random disordered state of particles, whereas, in the Big Crunch phase, randomly distributed particles are drawn into an order. Randomness can be seen as equivalent to the energy dissipation in nature while convergence to a local or global optimum point can be viewed as gravitational attraction. Since energy dissipation creates disorder from ordered particles, the random nature of the Big Bang (energy dissipation) is used as a transformation from an ordered state (a convergent solution) to a disordered or chaotic state (new set of candidate solutions).

   In order to demonstrate the efficiency of the BB–BC, it has been applied to the design of two water distribution networks (Hanoi, Double Hanoi). It is clear from the results that the proposed method can obtain higher quality solutions with better computation efficiency than some of the well known heuristic techniques. The comparison of the BB–BC results with those of the other algorithms shows the robustness of the present algorithm. Application of the BB–BC to the water distribution networks optimization is still in its infancy and further improvements is necessary. Although the BB–BC performs well in the exploitation, there are some problems in the exploration stage. Therefore, development of algorithmic strategies to improve computational efficiency and quality is likely required for designing practical or large water distribution networks.

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