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# COLLAPSE ANALYSIS BY ENDURANCE TIME METHOD

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## ABSTRACT

Estimation of collapse performance is primarily conducted through Collapse Fragility Curves (CFC's). The EDP-based approach is the main scheme for attaining such curves and employs IDA. Obtaining CFC's from IDA results is tremendously time consuming and computationally demanding. Introduction of more efficient methods of seismic analysis, can potentially improve this issue. The Endurance Time (ET) method is a straightforward method for dynamic analysis of structures subjected to multilevel excitation intensities. In this paper, collapse analysis using ET analysis results to obtain EDP-based CFC's, has been explained and demonstrated by a model. For verification, the resulting CFC has been compared to that obtained by IDA.

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KEY WORDS: collapse, collapse fragility curve, edp-based approach, incremental dynamic analysis, endurance time method, nonlinear time history analysis

## 1. INTRODUCTION

Protection against collapse has always been a major objective of seismic design. Collapse of a building during and shortly after an earthquake is the consequence of loss of the building's structural system integrity due to excessive deformation or force demand initiated in one, or several, component(s) of the building's structural system [1]. Excessive seismic demand triggers strength and stiffness deterioration in structural components and can lead to a partial or complete (global) collapse of the building.

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Global collapse may have several causes. The spread of an initial local failure from element to element may result in cascading or progressive collapse in different type of structures [2-4]. Incremental collapse occurs when the lateral displacement of an individual story is very large, and P-Δ effects fully diminishes the story shear resistance.

Collapse in this context is defined as the loss of lateral load-resisting capability of a building's structural system caused by ground shaking and usually triggered by large story drifts, which are amplified by P-Δ effects and deterioration in strength and stiffness of the components of the system [1]. According to this definition, the ability to model strength and stiffness deterioration of structural components when subjected to cyclic loading is a key issue in collapse evaluation of a building.

The inherent random nature of ground motions and the fact that no numerical model can fully characterize all building characteristics, adds more sophistication to the already complex problem of building collapse prediction under seismic excitations. As one of the major approaches towards collapse evaluation, probabilistic approaches describe a buildings' collapse potential as a probability of collapse [5-7]. In these approaches, possible sources of variability are identified and implemented in the process of collapse evaluation. The fundamental assumption in probabilistic approaches is that in estimating the collapse potential of a building, the effect of randomness in ground motion characteristics is independent of the uncertainty in analytical tools employed for earthquake hazard analysis and structural modeling.

Estimation of collapse performance requires the relation between a ground motion intensity measure (IM) and the probability of collapse, denoted as collapse fragility curve (CFC). The collapse fragility curve is the main and up to know, most reliable tool for collapse evaluation of structures. The EDP-based approach is currently one of the main methods for estimating the collapse fragility curve of a building. In this approach, seismic demand ($EDP_d$) and associated seismic capacity ($EDP_c$) are estimated probabilistically. These parameters are then used to calculate the probability of collapse given IM,P[C|IM], as the probability that the $EDP_d$ exceeds $EDP_c$ as shown below (1):

$$P[C|IM = im_i] = P[EDP_d \geq EDP_c \mid IM = im_i] = \Sigma_{all\ edpc}\, P[EDP_d \geq EDP_c \ |EDP_c = edp_{ci}\,, IM = im_i]P[EDP_c = edp_{ci}] \quad (1)$$

In this equation, $P[EDP_d \geq EDP_c \mid EDP_c = edp_{ci}\,, IM = im_i]$ is the probability that the demand exceeds the capacity value $edp_{ci}$ at $IM=im_i$ and $P[EDP_c=edp_{c\,i}]$is the probability that capacity is equal to $edp_{ci}$ .The procedure of obtaining the CFC of a structure from IDA curves is elaborated in [1].

Despite of outstanding advances in computer technology in recent years, because of the high number of dynamic analyses involved, from linear behavior all the way to extreme levels of structural nonlinearity and collapse, the process of obtaining collapse fragility curves from IDA results, is tremendously time consuming and computationally demanding. Thus collapse evaluation of actual structures by employing IDA results, is practically and economically not feasible. Introducing more simple and efficient methods of seismic analysis, can be a solution to this problem. The Endurance Time (ET) method is an innovative and straightforward method for dynamic loading and analysis of structures, apprehensible for standard level of seismic engineering knowledge. The basic idea of ET method was originally introduced by Estekanchi et al [8]. The concept of ET method is similar to the exercise test used by cardiologists for assessing the condition of cardiovascular system of patients [9]. In this novel procedure, an intensifying artificial accelerogram, termed as Endurance Time Acceleration function or ETA, is applied to the structure and its various structural responses monitored. Since different times in ET acceleration functions corresponds to different seismic intensities, a single ET time history analysis provides structural response information at the linear elastic level, up to extremely high levels of nonlinearity and finally collapse of a structure, thus significantly reducing time and computational cost compared to IDA [10].

In the following, the procedure of obtaining collapse fragility curves using ET analyses results is explained. This process is demonstrated on a 3-story 1-bay steel moment frame. At the end, to verify the process, the resulting ET based CFC of the model is compared to that obtained by IDA.

## 2. ET DEMAND CURVES

In this section, the procedure for obtaining plots of an Engineering Demand Parameter (EDP) versus a measure of seismic intensity (IM). In IDA analyses, the relation between EDP and IM is termed as an ‘IDA curve’[9]; analogous to this denomination, this relation is termed as ‘ET demand curve’ for ET analysis results, i.e. (EDP, IM) data pairs. This process is demonstrated on a 3-storey 1-bay steel moment frame, with fundamental period and design base shear coefficient equal to 0.72 (sec) and 0.15, respectively. This frame is termed as model F1 from now on. A schematic presentation of model F1 and section properties of its members are illustrated in Figure 1 and Table 1, respectively.

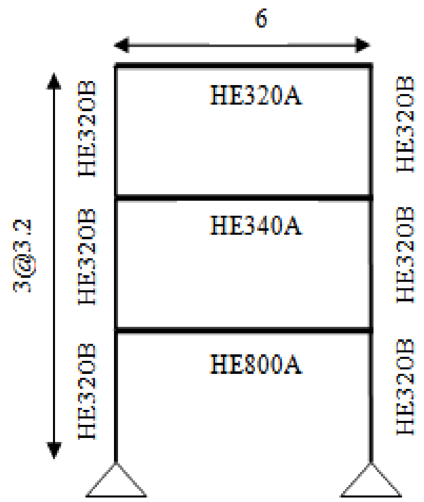


Figure 1. Sample model F1 geometry (dimensions in meters)

Table 1. Section properties of model F1 members

| Section | HE320B | HE320A | HE340A | HE800A |
|---|---|---|---|---|
| Area | 0.0161 | 0.0124 | 0.0133 | 0.0286 |
| d | 0.32 | 0.31 | 0.33 | 0.79 |
| $b_f$ | 0.30 | 0.30 | 0.30 | 0.30 |
| $t_f$ | 0.0205 | 0.0155 | 0.0165 | 0.0280 |
| $t_w$ | 0.0115 | 0.0090 | 0.0095 | 0.0150 |
| $I_x$ | 3.082E-04 | 2.293E-04 | 2.769E-04 | 3.034E-03 |

**Note: Values are in SI units**

At the beginning, ET analysis should be conducted. In order to more likely observe the building's behavior and responses up to its collapse point, ET acceleration functions with a longer time duration of around 40 seconds is used for ET analysis. For this reason, the series g ET acceleration functions, i.e. ETA40g (1, 2, 3), is employed for ET analysis of model F1. In development of the ET series g acceleration set, the ASCE-7 [12] design spectrum (for L.A. with the following design parameters: $S_s$=1.5; $S_1$=0.6; $F_a$=1.0; $F_v$=1.3; $T_L$=8) is taken as the target spectrum, i.e. response spectrum of series g ETA's match this spectrum at the target time ($t_{Target}$ = 10 sec). ETA40g01 acceleration function is depicted in Figure 2. In the analyses, one horizontal component of the acceleration functions has been considered and dynamic soil-structure interaction was neglected. P-Δ effects have been considered.

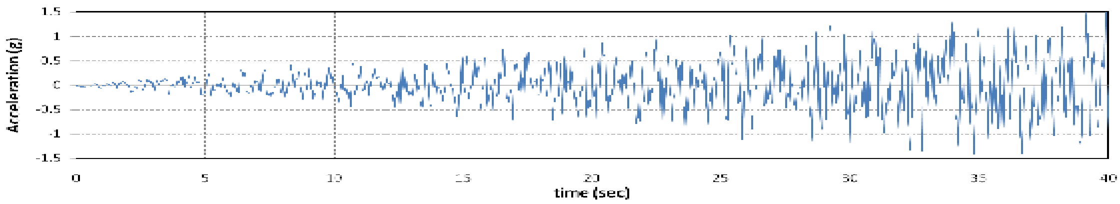


Figure 2. ETA40g01 accelerogram

ET analysis results are presented by increasing ET curves where the *y* coordinate at each time value, t, corresponds to the maximum absolute value of the required EDP in the time interval [0, t] as given in Eq. (2).

$$\Omega\, (\, f(t)\, ) \equiv\ Max\, (\, Abs(f(\tau)) : \tau\, (\, [0\, ,\, t]\ \ ) \qquad (2)$$

In this equation Ω is the Max_Abs operator as defined above and f(t) is the desired response history such as base shear, interstory drift ratio, a damage index or other parameter of interest. The *x* coordinate axis of an ET curve is time, which is correlated with the intensity measure (IM).

After performing ET analyses, as also necessary for IDA based collapse analysis, plot of an appropriate Engineering Demand Parameter (EDP) versus a measure of seismic intensity (IM), is required. Selecting a suitable EDP or IM, depends on the application and the structure itself. For structural damage of shear buildings, Vamvatsikos and Cornell [11] state that the maximum interstory drift ratio (maxIDR) relates well to joint rotations and both global and local story collapse, thus becoming a strong EDP candidate, and the 5 percent damped spectral acceleration at the first or fundamental mode, Sa($T_1$, $\xi$= 5%), is a suitable parameter for IM. Thus for model F1, maxIDR and Sa ($T_1$, $\xi$= 5%) are chosen as the EDP and IM, respectively.

To obtain the ET demand curve, first, the maximum interstory drift ratio (maxIDR) ET curve, i.e. plot of maximum absolute values of maxIDR versus time, is derived so the EDP (maxIDR) values are in hand. In figure 3, maxIDR time histories along with corresponding ET curves for the 3 accelerograms in the ETAg set, i.e. ETAg1, ETAg2 and ETAg3, for model F1 is presented.

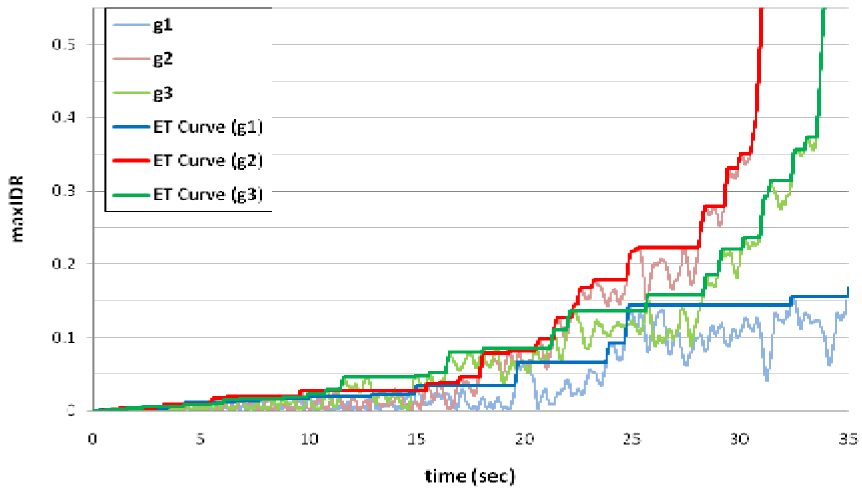


Figure 3. maxIDR time histories and ET curves for model F1 subject to ETAg set

As mentioned before, the time parameter in ET analysis corresponds to level of seismic intensity, so with a proper scheme, the time parameter can be converted to corresponding IM values, thereupon resulting to the ET demand curve. This conversion is obtained using the fact that the response spectrum of the ETAg acceleration functions, match the ASCE-7 design spectrum at the target time. Thus, different IM values can be calculated at different time steps using the following equation:

$$S_{aT}(T,t) = \frac{t}{t_{Target}} \times S_{aC}(T) \tag{3}$$

Where,

$S_{aT}$ (T,t) : target acceleration response at time t or in the present context, IM.

T : first mode period of vibration of the structure

$S_{aC}$ (T) : design acceleration spectrum; in this project, the ASCE-7 design spectrum.

It is important to note that in any type of dynamic time history analysis such as IDA, the ground motion set to be used must be selected carefully. The selected earthquake records should be compatible, i.e. in soil conditions, fault rupture mechanism, seismic wave propagation and so on, with the seismic design spectrum or site conditions, of the structures to be analyzed. At ideal condition, the chosen acceleration set's characteristics are completely compliant to the site of interest's conditions. In this case, the IDA curves of all the earthquake records in the selected set are completely identical, i.e. predicting similar response values at each IM. Thus any difference among the IDA curves, origin from poor selection of earthquake records. This criterion can also be applied to ET acceleration functions. Since ET acceleration functions are artificial computer-generated accelerograms, different acceleration functions in each ETA set (such as ETAg1, ETAg2 and ETAg3 in series g ETA set) is a result of weakness in the optimization method employed to generate these ET acceleration functions. Thus similar to IDA, in the ideal case (a complete and flawless optimization technique to develop the ETA functions), the ETA's generated are completely similar, leading to completely identical ET demand curves.

Figure 4 presents the ET demand curves (abbreviated 'ETD' in the figure) for model F1, obtained from the ETAg acceleration set, i.e. ETAg1, ETAg2 and ETAg3 acceleration functions.

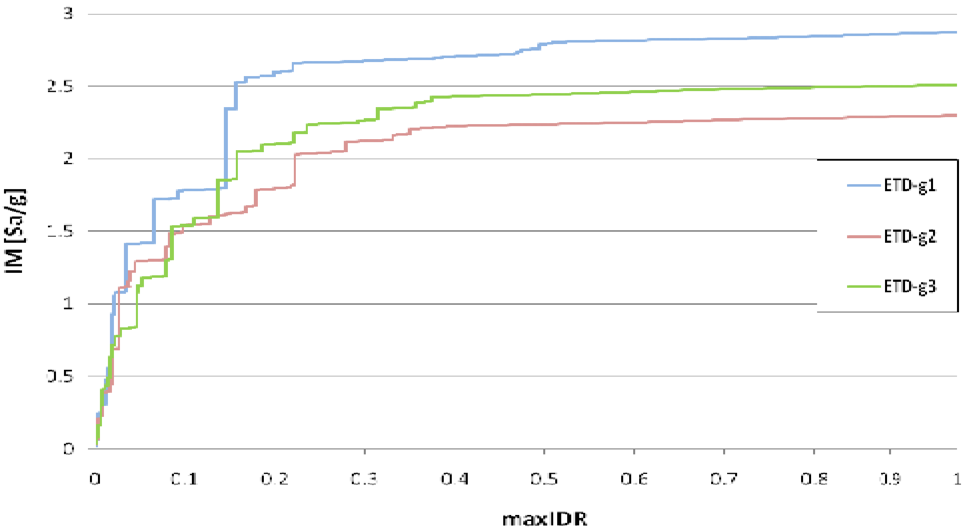


Figure 4. ET demand curves of model F1

## 3. COLLAPSE FRAGILITY CURVES

In this section, EDP-based collapse fragility curves are obtained from ET demand curves. This process is conceptually similar to the procedure of attaining CFC's from IDA curves. Since this process has been thoroughly explained in previous literature [1], to avoid complexity, elaboration of this process is omitted and only exclusive notes to ET results are mentioned.

As a main part of the process for obtaining collapse fragility curves in the EDP-based approach, the demand and capacity cumulative histograms should be initially determined. Demand cumulative histograms are specified by intersecting horizontal lines (corresponding to definite levels of IM) with IDA/ ET demand curves (indicated by squares for IM=2gin figure

5) and plotting the cumulative distribution of the attained data points (for model F1, maxIDR demand values). The capacity cumulative histogram is determined for a set of IDA/ET demand curves, defined as the EDP value at which the slope of the IDA/ET demand curve exceeds 20% of the median of the initial (elastic) slope of all IDA/ET demand curves for the last time [13]. These capacity values are indicated by circles in figure 5. The demand and capacity histograms are also plotted in figure 5; in grey and black respectively. So to implement the EDP-based approach for ET analysis results, capacity and various demand cumulative histograms should be first determined. As can be seen in figure 4, ET demand curves, as like all ET curves, have a graduated and step-like shape. Since the serrated nature of ET demand curves produces error and irregularity in evaluating capacity and demand cumulative histograms; necessary for collapse fragility curve calculation, each ET demand curve is smoothed by a curve smoothing technique. For ET demand curves of figure 4, the robust locally weighted scatter plot smoothing (robust loess) method, is utilized. In figure 5, original and smoothed ET demand curves of model F1 and a schematic presentation of the EDP-based approach is illustrated.

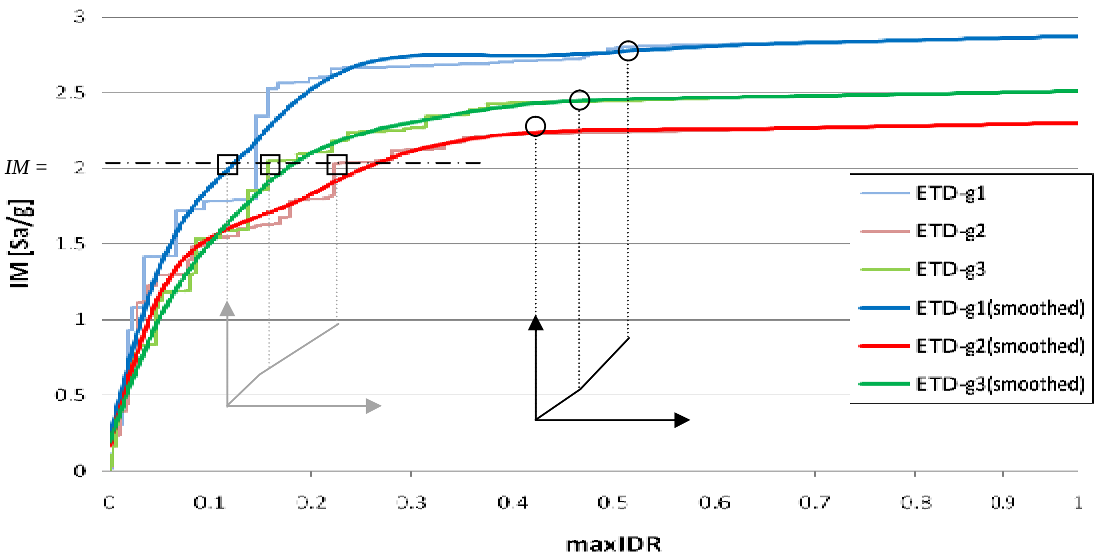


Figure 5. Original and smoothed ET demand curves of model F1

After attaining smoothed ET demand curves, the aforementioned cumulative histograms for demand and capacity in the EDP-based approach are estimated and equation 1 is evaluated numerically to find the probability of collapse given IM at various IM levels (P(C|IM) data points). The collapse fragility curve, using the EDP-based approach, is obtained by fitting a log-normal distribution [5], or any other appropriate function, to the probability of collapse given IM data points. P(C|IM) data points and resulting collapse fragility curve (log-normal function fit) for model F1 is presented in Figure 6.

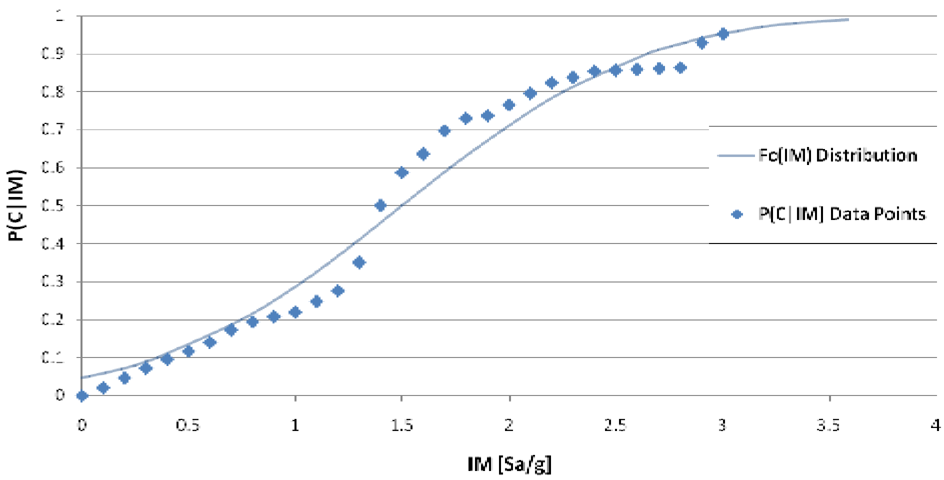


Figure 6. EDP-based P (C|IM) data points and collapse fragility curve for model F1

In the following, to validate the ET collapse fragility curve process, IDA is conducted on model F1 and the corresponding CFC obtained by IDA curves is compared to the model's ET-based collapse fragility curve.

## 4. COMPARATIVE STUDY

Seven ground motions were selected for IDA. They were chosen to be compatible, i.e. in soil conditions, fault rupture mechanism, seismic wave propagation and so on, with the seismic design spectrum of the model. These earthquake records were selected from a set of 20 ground motions, used in the ATC-55 project (FEMA-440 report) [14], which are recorded on stiff soil (class C) conditions. For convenience, this set of 7 ground motions is named the GM set from now on. The GM set, along with their general characteristics, are summarized in Table 2.

Table 2. Description of the GM set

| No | Date | Earthquake name | Record name | Magnitude (Ms) | Station number | PGA (g) |
|---|---|---|---|---|---|---|
| 1 | 01/17/94 | Northridge | NRORR360 | 6.8 | 24,278 | 0.51 |
| 2 | 06/28/92 | Landers | LADSP000 | 7.5 | 12,149 | 0.17 |
| 3 | 04/24/84 | Morgan Hill | MHG06090 | 6.1 | 57,383 | 0.29 |
| 4 | 10/17/89 | Loma Prieta | LPAND270 | 7.1 | 1,652 | 0.24 |
| 5 | 10/17/89 | Loma Prieta | LPGIL067 | 7.1 | 47,006 | 0.36 |
| 6 | 10/17/89 | Loma Prieta | LPLOB000 | 7.1 | 58,135 | 0.44 |
| 7 | 10/17/89 | Loma Prieta | LPSTG000 | 7.1 | 58,065 | 0.50 |

In order to be consistent with seismic codes, the GM set is scaled. For this purpose, they are scaled according to ASCE-7guidelines, i.e. scaled such that their 5%-damped linear spectral acceleration response is equal or greater than the ASCE-7 design spectrum, for the period range of $0.2T_1$ to $1.5T_1$, where $T_1$ is the fundamental period of vibration of each frame modeled as a linear system. This spectrum is also used as the target spectrum in developing the ETAg acceleration set, thus being compliant to the ET analysis acceleration functions employed. Similar to ET analysis conditions, only one horizontal component of the ground motions has been used, dynamic soil-structure interaction was neglected and P-Δ effects have been considered. Once again, the maximum interstory drift ratio and the 5 percent damped spectral acceleration at the first or fundamental mode, $Sa(T_1, \xi= 5\%)$, are taken as EDP and IM respectively. After IDA, resulting IDA curves of the GM set for model F1 is attained (Figure 7).

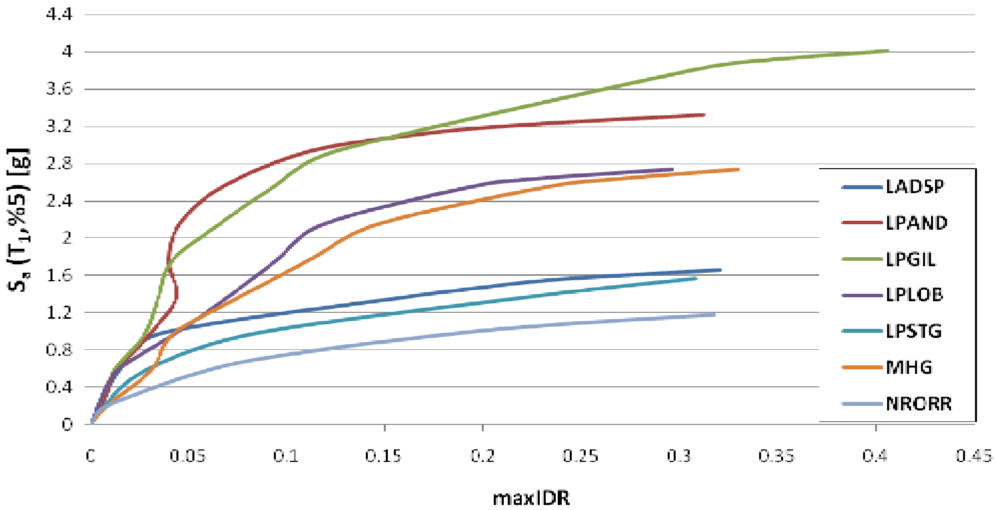


Figure 7. IDA curves of GM set for model F1

Before proceeding any further, the important question is how the results of ET and IDA analyses can be compared so to obtain comparable collapse fragility curves. Results of ET analysis are obtained through time and as mentioned before in this method, the time is correlated with intensity measure (IM). Therefore, different values of Engineering Demand Parameters (EDP) are calculated for different values of IM in an ET analysis. To establish a relation between the results of the ET method and any other method, the IM value of the other method should be found in the ET analysis. Therefore, a procedure should be defined to find an equivalent time in the ET analysis in which the IM values of the two methods are equal. For this purpose, Eq.(4) is used to correlate the IDA scaling factor ($S_1$) of the earthquake records with the endurance time of the specific structure via an equivalent time:

$$t_{ET} = t_{Target} \times S_1 \times S_2 \tag{4}$$

Where $t_{ET}$ is the equivalent time in ET analysis which corresponds to an IDA analysis with ground motions scaled by the $S_1$ scale factor, and $S_2$ is the correction factor defined as the ratio between the acceleration response spectrum of a specific record and the target spectrum at the fundamental period of vibration. This factor is calculated using Eq. (5):

$$S_2 = \frac{S_{aS}(T_1)}{S_{aC}(T_1)} \tag{5}$$

Scale factors, $S_2$, corresponding to each record of the GM set, and the average $S_{2,Ave}$ for model F1is shown in Table 3.

Table 3. $S_2$ scale factors

| LADSP | LPAND | LPGIL | LPLOB | LPSTG | MHG | NRORR | $S_{2,Ave}$ |
|---|---|---|---|---|---|---|---|
| 1.36 | 1.37 | 1.09 | 1.05 | 1.86 | 1.96 | 2.11 | 1.54 |

In table 4, maximum interstory drift values obtained by IDA and ET analyses for model F1 are compared. These results are the average values of the GM ground motion set and series g ET acceleration functions for IDA and ET analyses, respectively.

Table 4. Comparison of ET and IDA maximum interstory drift values for model F1

| IDA scaling factor (S1) | Equivalent time (sec) | IM [Sa/g] | maxIDR | |
|---|---|---|---|---|
| | | | ET | IDA |
| 0.2 | 3.08 | 0.2217 | 0.0061 | 0.0059 |
| 0.6 | 9.24 | 0.6651 | 0.0303 | 0.0257 |
| 1 | 15.40 | 1.1085 | 0.0732 | 0.0674 |
| 1.4 | 21.56 | 1.5518 | 0.1442 | 0.1397 |
| 1.8 | 27.72 | 1.9952 | 0.1584 | 0.1471 |

After obtaining comparable results, i.e. EDP or maxIDR values at similar IM's for ET and IDA analyses, the IDA-based collapse fragility curve is attained for model F1. ET and IDA based collapse fragility curves for model F1 is plotted in Figure 8.

As can be seen, the two CFC's slightly differ and ET predicts lower seismic collapse capacity than IDA, i.e. at each certain IM, the ET collapse fragility curve shows a higher probability of collapse than IDA's. This difference can be mainly due to the fact that the 20% collapse capacity criteria, is dependant on the slope or general shape of IDA/ET demand curves. Since ET demand curves, unlike IDA curves, were initially smoothed before use for obtaining the collapse capacity of the structure, thus the collapse capacity calculated in this way is very sensitive to the smoothing technique employed. Therefore the smoothing technique used completely affects this parameter and consequently the resulting fragility curve. In IDA, the IDA curves were obtained by interpolating between successive data points, thus preserving the main shape of the IDA curve.

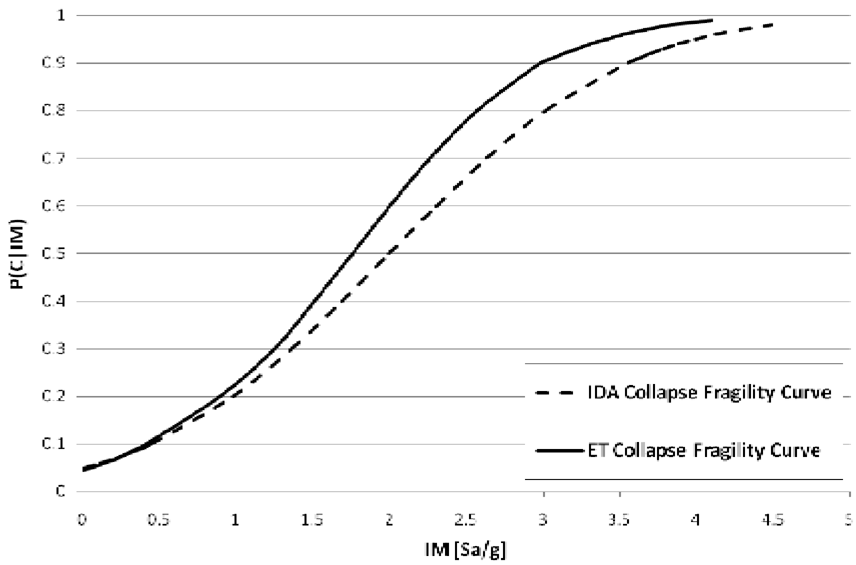


Figure 8. ET and IDA obtained collapse fragility curves of model F1

## 5. SUMMARY AND CONCLUSIONS

In this paper, application of collapse analysis using ET analysis results to obtain EDP based Collapse fragility curves (CFC's), from obtaining ET curves for different response parameters to ET demand curves, and all the way up to attaining Collapse fragility curves, has been explained and demonstrated. Generally, it can be seen that ET has no limitations for this purpose and similar to IDA, can be employed as an effective tool for collapse analysis of various structures. It should be noted that alike for IDA curves, to reach trustworthy and reliable CFC's, ET demand curves should be continued up until a point where dynamic instability of the considered structure is reached. This goal is mainly translated into the ET demand curve developing into a flat plateau in this study, i.e., where significant increase in seismic demand is observed at a particular seismic intensity.

To demonstrate the procedure, a 3 story 1 bay steel moment frame was analyzed by the ET method and its resulting collapse fragility curve obtained and compared to its IDA based fragility curve. It can be seen that although ET predicts a poorer collapse performance (an average 20% higher probability of collapse at different seismic intensities); considering the tremendous reduction in computational effort and cost resulted by ET, this level of variation among the CFC's has been considered as acceptable in this research.

## NOMENCLATURE

$EDP$ = Engineering Demand parameter
$EDP_c$ = Seismic Capacity
$EDP_d$ = Seismic Demand

$IM$ = Intensity Measure
P(C|IM) = probability of collapse at seismic intensity level of IM
$S_1$ = Seismic intensity scaling factor in IDA
$S_2$ = Correction factor, $S_2 = \dfrac{S_{aS}(T_1)}{S_{aC}(T_1)}$
$S_a$ = Acceleration response
$S_{aC}(T)$ = Code acceleration response for period $T$
$S_{aS}(T)$ = Scaled ground motion record response for period T
$S_{aT}$ (T,t) = Target acceleration response at time t.
$t$ = Time
$T_1$ = Fundamental period of vibration (sec)
$t_{ET}$ = Equivalent time
$t_{Target}$ = Target time
$\tau$ = Dummy variable for time
$\Omega$ = Max_Abs operator

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